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An Investigation of Factors Affecting Student Performance in Algebraic Word Problem Solutions

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An Investigation of Factors Affecting Student Performance
in Algebraic Word Problem Solutions

By
Jerry Eugene Wright

A Dissertation Submitted to the
Gardner-Webb University School of Education
in Partial Fulfillment of the Requirements
for the Degree of Doctor of Education

Gardner-Webb University
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Approval Page

This dissertation was submitted by Jerry Eugene Wright under the direction of the persons listed below. It was submitted to the Gardner-Webb University School of Education and approved in partial fulfillment of the requirements for the degree of Doctor of Education at Gardner-Webb University.

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Abstract

An Investigation of Factors Affecting Student Performance in Algebraic Word Problem Solutions. Wright, Jerry Eugene, (2014): Dissertation, Gardner-Webb University, Algebra Word Problem/Problem Solving/Textual Translation/Algebra/Common Core State Standards

This dissertation was designed to investigate the potential significance of several student algebraic word problem solving skills. The solution of an algebra word problem (AWP) requires the creation and solution of an equation based on the problem context. Common Core State Standards in both English Language Arts and Mathematics emphasize student learning and proficiency in algebra word problem contexts. Eight factors of student mathematical ability were proposed, and three of those eight factors were studied in depth to determine their significance. Similar research supported the theory that the *translation phase* of the solution process presented the student with the most significant difficulty, as the natural language of the problem statement was changed into mathematical symbolism and equations. The findings of the current research suggested that additional cognitive tasks and abilities were required to obtain successful solutions to AWP, in addition to mere *translation*.

Students enrolled in Algebra I, Algebra II, and Pre-Calculus courses in public secondary schools were surveyed about their previous and current experiences in solving AWP and were given a battery of assessments to determine individual performance levels in solving AWP. Data on perceived difficulties in AWP solution, as mentioned by students on mathematical learning style surveys and on the battery of assessments for actual AWP solution performance, were collected. Measures of association and correlation were calculated and ANOVA analyses were conducted.

Statistically significant rank-order correlations were found in comparisons of participant performance in a) ability to identify algebraic operations, b) ability to recognize relational statements, and c) ability to translate text into equations when compared with correctly solved AWP scores. Statistically significant differences in mean number of correctly solved AWP were found between grade levels 9, 10, and 11 and also between the following courses: Algebra I, Algebra II, and Pre-Calculus. No statistically significant differences were found in comparison of participant gender groups and no statistically significant differences were found in comparison of participant ethnicity groups. Criteria for identification of a participant as a *proficient AWP solver* were developed. Participant mathematical learning style characteristics were investigated to determine influential factors in measured AWP solution ability.

Table of Contents

	Page
Chapter 1: Introduction	1
Statement of the Problem.....	3
Specific Research Goals	6
Types of Algebraic Word Problems	6
General Discussion of the Eight Factors.....	8
Reading Comprehension (Factor 1).....	10
Identifying Mathematical Operation Clues (Factor 2).....	11
Recognition and Use of Geometry Sketches (Factor 3).....	14
Recognizing Relational Statements (Factor 4)	16
Formula Selection (Factor 5)	19
Translation of Text and Equation Writing (Factor 6)	19
Equation Solving (Factor 7).....	20
Checking Solutions (Factor 8)	22
Additional Factors.....	22
Chapter 2: Literature Review.....	24
Overview.....	24
Research Questions	24
Identifying Mathematical Operation Clues.....	25
Recognizing Relational Statements	27
Translation of Text and Equation Writing.....	31
Chapter 3: Methodology	40
Introduction.....	40
Participant Characteristics and Sampling Procedures.....	40
Instrumentation	42
Pilot Study Data and Discussion.....	44
Research Design.....	47
Data Collection Procedures.....	48
Chapter 4: Results	51
Introduction.....	51
Instrument Validation	51
Descriptive Statistics for Participants	51
Research Questions Data and Analyses	53
Rank-order Correlation Analyses	55
ANOVA Comparisons of Subgroup CS Means	59
Characteristics of a Proficient AWP Solver.....	60
Participant Mathematical Learning Style Comparisons	67
Chapter 5: Discussion	70
Formal Research Questions Addressed	70

Discussion of ANOVA Results	75
Discussion of the Characteristics of a Proficient AWP Solver.....	79
Discussion of Participant Mathematical Learning Style Comparisons.....	81
Contributions to the Literature.....	84
Limitations	85
Delimitations.....	85
Considerations for Future Research.....	86
References.....	87
Appendices	
A Table A1: Examples of Typical AWP Taught in Algebra I and II.....	92
Table A2: Essential Skills and Procedures Used to Solve AWP.....	95
Table A3: Flesch Reading Ease Score Mapping Table	96
Table A4: AWP Used in Data Collection, Modified from Table A1	97
B Solutions to Algebra Word Problems	98
C Form A: School and Student Data Record Form for Research Participants.....	114
Form B: Student Mathematics Learning Style Questionnaire	116
Form C: Mathematical Operation Identification.....	118
Form D: Recognizing Relational Statements.....	119
Form E: Translating Written Text into Mathematical Sentences	121
Form F: Student AWP Solutions	123
D Form G: Student AWP Solution Scoring Rubric.....	124
E Table E1: Rank-order Correlations for 2-Variable Participant Subgroups.....	126
Table E2: Rank-order Correlations for 3-Variable Participant Subgroups.....	128
Table E3: Rank-order Correlations for 4-Variable Participant Subgroups.....	129
F Table F1: Statistics for Mean Correct Score <i>CS</i> , on Form F, by Participant Subgroups	130
Table F2: Statistics for Total Number Correct <i>OA</i> , on Form C, by Participant Subgroups	132
Table F3: Statistics for Total Number Correct <i>RA</i> , on Form D, by Participant Subgroups	133
Table F4: Statistics for Total Number Correct <i>TA</i> , on Form E, by Participant Subgroups	134
G Table G1: Distribution of PAWPS and Non-PAWPS Participant Responses, Form B	135
Table G2: Distribution of Participant Responses by Grade-level, Form B	138
Table G3: Distribution of Participant Responses Course, Form B.....	141
Tables	
1 Summary of High School Student Efforts on Equation and AWP Solution.....	2
2 Spearman Rank-Order Correlation Coefficients for Comparisons of <i>OA</i> , <i>RA</i> , and <i>TA</i> with <i>CS</i> values: Pilot Study Data.....	46

3	Participant Number and (Percentages) for Gender, Ethnicity, Course, and Grade Level	53
4	Statistics for Complete Participant Group Performance on Initial Assessments and AWP Solution Efforts	55
5	Spearman's Rank Order Correlation Coefficients for ALL and 1-variable Participant Subgroups	58
6	CS Values for Proficient AWP Solver by Subgroups.....	62
7	Statistics for Proficient AWP Solver Performance on Initial Assessments and AWP Solution Efforts	63
8	PAWPS Solution Strategy, Yes or No for Correct Solution, and CS values.....	65
9	AWP Solution Strategies and Correct Solution Status for ALL and PAWPS.....	67
10	Comparison of Subgroup MLSS Responses, indicated as Percents	84
Figures		
1	Course and Grade Composition Frequency, by Gender and Ethnicity	41
2	Boxplots of OA, RA, TA, and CS values for ALL and PAWPS Subgroups.....	64

Chapter 1: Introduction

Much has been said in current education about the need for students to exercise *critical thinking* and efficient *problem solving ability*. The current research addressed a problem-solving skills component within the curriculum of Algebra I, Algebra II, and Pre-Calculus courses, in particular, the solution of algebraic word problems (AWP).

Vernooy (1997) stated:

Word problems are part of the mathematics curriculum for a good reason: They illustrate the connection between mathematics and clear, critical thinking on any subject. Word problems emphasize the precise definitions of terms, the making of only those assertions which specifically apply to the issues or objects under discussion, and the application of careful reasoning in problem solving. These are all vital skills in any intellectually challenging profession, in forming thoughtful judgments about political and educational issues, and in making personal decisions. (p. 5)

AWP were defined within this study as presenting a problem statement consisting of one or more sentences having some known or unknown values, with explicitly or implicitly stated relationships between the values. AWP were particularly well suited for the current investigations due the essential requirement that the student exercise skills in addition to rote manipulative algebraic procedures to arrive at a solution. AWP are an integral part of algebraic curriculum and instruction, and the improvement of student ability to solve AWP is a critical concern. A common thread of difficulty observed within the high school student population is that students are significantly less proficient at solving AWP than any other algebraic task. This sentiment was voiced by Lester,

Garofalo, and Kroll (1989) as they stated, “For generations, mathematics teachers have voiced concern about the inability of their students to solve any but the most routine verbal [or written-word] problems despite the fact that they seem to have mastered all the requisite computational skills and algorithmic processes” (p. 1).

Assessment data for student performance in solving standard algebraic equations and completing AWP were submitted by three local high school mathematics teachers. The students were enrolled in two regular-level Algebra II classes and one honors-level Algebra II class. The students were given several assessments of equation and/or AWP solving proficiency. Table 1 displays the number of equations attempted, number of correct solutions, and the percent of correct responses for standard equation solutions and AWP solutions. Partial credit was awarded for solution attempts on AWP but not for standard equation solution attempts. Students in all classes were able to solve standard equations at the 84% correct level, but fewer than 68% of the AWP solutions were correct (James Davis, personal communication, April 8, 2013).

Table 1

Summary of High School Student Efforts on Equation and AWP Solution

Algebra II Course	Standard Equation Solution			AWP Solution		
	# Equations	Average # Correct	Percent Correct	# AWP	Average # Correct	Percent Correct
Regular 1	9	8.1	90.0%	2	0.7	35.0%
Regular 2	11	9.3	84.5%	3	1.0	33.3%
Honors	16	13.7	85.6%	4	3.0	67.5%

The data presented above suggested that students experienced less success in

solving AWP than they did in solving equations. This may be due to an imbalance in instruction between the teaching of problem set-up and problem solution, as indicated by Aziz, Pain, and Brna (1995), who asserted, “The major difficulty [in AWP solution] lies in the translation [or set-up] phase although most instruction focuses on the solution phase” (p. 2). Additionally, Ilany and Margolin (2010) suggested that the students’ difficulty in solution of AWP, as compared to rote equation solution, was compounded by the existence of “the knowledge gap between mathematical language and natural language, and knowledge gaps between the textual unit and the hidden mathematical structure” (p. 141). Hegarty, Mayer, and Green (1995) suggested that significantly more cognitive processing of mathematical knowledge occurred as the student works from verbal or written text to solve an AWP than from solving standard equations devoid of context. A scrutiny of the chapter review exercises of the first seven chapters of the Algebra II textbook by Larson et al. (1995) indicated that 16 out of the 367 exercises are AWP, in the sense used in the current research. This represented approximately 4.36% of the total problems in which the student in Algebra II was engaged in learning. Within the content of Algebra I and Algebra II courses, there are at least one dozen different types of AWP varying in complexity, content, and applicable solution strategies. A selection of AWP (Appendix A) was used in this study.

Statement of the Problem

As stated in *Principles and Standards for School Mathematics* (NCTM, 2000): Solving problems is not only a goal of learning mathematics but also a major means of doing so . . . By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations (p. 52).

Students enrolled in mathematics courses at or above the Algebra I level have typically encountered a variety of AWP, which differ in the complexity of the problem and the necessary solution procedures. The solution of such problems required the following: reading comprehension, recognition of implied or stated operations, identification and quantification of stated or implied relationships within the AWP, selection and use of visual aids such as sketches, translation of written language into symbolic equation form, and application of learned algebraic manipulative procedures to solve one or more equations. In Algebra I and Algebra II courses, the student has learned several manipulative procedures such as simplification, evaluation, substitution, distribution, and equation solving. Most students can master these procedures with due diligence and practice and appropriately apply them in the solution of many simple equation problems. The solution of an AWP required the application of the aforementioned manipulative skills, but only after mathematical equations or inequalities appropriate to the context had been written. The eight *factors* proposed by the researcher were: a) ability to read and comprehend written text in the word problem, b) ability to identify written clues indicative of mathematical operations, c) ability to identify geometric objects referenced in the written text and to draw an appropriate sketch to match the problem statements, d) ability to recognize relational statements between component parts of written text, e) ability to select the applicable formula based on the problem context, f) ability to translate written text into mathematical equations, g) ability to apply manipulative problem solving procedures to arrive at a correct response, and h) ability to distinguish between correct and incorrect solutions based on problem constraints. The student began the AWP solution task with the reading of one or more written statements framed within a *real-life problem* context. The current research

connected to student learning goals of the recently enacted *Common Core State Standards Initiatives for English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects* (ELAL) (2010) as well as *Common Core State Standards Initiative for Mathematics* (MATH), which emphasized student learning and proficiency in word problem contexts. As mentioned in the ELAL document, the student must

read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence . . . to support conclusions drawn from the text . . . [and] interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning. (ELAL, 2010, p. 60)

Furthermore, the emphasis within science or technical subjects is for students in Grades 9-12 to become proficient at “determining the meaning of symbols, key terms, and other domain specific words and phrases as they are used in a specific scientific or technical context” (ELAL, 2010, p. 62).

The MATH (2010) document made reference to several aspects of word problem solving in 20 separate standards for Grades K-12. For example, standard 3.OA.D.8 required the student to be able to “solve two-step word problems using the four operations . . . and to represent these problems using equations with a letter standing in for the unknown quantity” (MATH, 2010, p. 23). Standard 6.EE.A.2 required the student to “write, read, and evaluate expressions in which letters stand for numbers” (MATH, 2010, p. 43). Standard 6.EE.B.7 required the student to be able to “solve real-world and mathematical problems by writing and solving equations” (MATH, 2010, p. 44). Standard 6.EE.C.9 required the student to be able to “use variables to represent two

quantities in a real-world problem that change in relationship to each other; i.e. $d=65t$ ” (MATH, 2010, p. 44). These standards suggested the importance of a renewed emphasis upon student proficiency at navigating the difficult terrain of AWP solution processes.

Specific Research Goals

The specific research goals were to determine a) a measure of correlation between student performance in the ability to identify written clues indicative of mathematical operations and overall problem-solving performance on an AWP, b) a measure of correlation between student performance in the ability to recognize relational statements between component parts of written text and overall problem-solving performance on an AWP, and c) a measure of correlation between student performance in ability to translate written text into mathematical equations and overall problem-solving performance on an AWP. Additionally, measures of qualitative association between student mathematical learning-style responses and overall problem-solving performance on AWP data on student experience with solving algebraic word problems were calculated, and data on student mathematical learning-style and student gender were collected to facilitate the analyses. The data collection involved students in secondary school level Algebra I, Algebra II, and Pre-Calculus courses.

Types of Algebraic Word Problems

Students encounter AWP as content within Algebra I, Algebra II, and Pre-Calculus mathematics courses in U.S. middle or secondary schools. The primary differences in AWP statements and solution procedures between the courses are the number of variables used in the equations, the number of equations used, and the methods of solution. Examples of 15 typical AWP are provided (Appendix A, Table A1), and the corresponding researcher-prepared solutions are included (Appendix B). The prepared

solutions were analyzed to determine similarities within the problem types with respect to the skills and procedures employed. A summary is presented in Appendix A, Table A2. The information in the table has been compiled to facilitate the research investigation of student-centered factors related to problem comprehension, model construction, and algebraic solution skills. Similarities and differences between essential student skills and procedures may signal significant factors bearing on eventual AWP solutions. A built-in feature of Microsoft Word (2010 version) was used to determine the Flesch-Kincaid Grade Level (FKGL) readability and Flesch Reading Ease (FRE) scores for each problem statement (Appendix A, Table A2). The FKGL scores are standard U.S. grade-school level values. The FRE scores rate text on a 100-point scale based on the average number of syllables per word and words per sentence. The FRE scores are identified in Appendix A, Table A3. A discussion and analysis of the skills the student has to employ in order to successfully and efficiently solve each problem follows from study of Table A2 (Appendix A). Only four of the AWP are on a grade 9.0 or higher level as seen in the FKGL scores. Two problem types (MIXTURE and ALLOY) are on a post-high school reading level, two others (COSTS and INVESTMENT) are on a middle-school reading level, and the remaining types are below a sixth grade reading level. The need for a sketch or figure is primarily determined by the geometric or motion-involved nature in six problems. The use of a formula in ten problems is either an established mathematical one, such as "*distance = rate x time*" or one from a related subject, such as "*revenue = unit cost x number sold.*" The use of a table for organizing information is useful in four problems. Finally, one equation is used in six problems, whereas all others require two equations. The equations differed in complexity, with linear, quadratic, and proportion equations being used. One-variable equations were used in three problems. All 15

problems required the student to identify textual clues indicating operations to be used and to identify relationships between quantities within the problems; the clues and relationships were explicitly stated or implied. In addition, all 15 problems required the student to write and solve one or more equation(s). The review of the literature focused on recent research on the student-centered factors related to problem comprehension, model construction, and algebraic solution skills.

General Discussion of the Eight Factors

The comment that solving an AWP is difficult was stated in nearly one dozen references including the following: “Word algebra problems are hard. Even students who are good at mathematics often hate them, and frequently neither teacher nor textbooks know how to teach them” (Weaver & Kintsch, 1992, p. 419). Vernooy (1997) also commented,

Some students are going their entire grade school and high school careers without being required to learn to do word problems. Even their teachers tell them that ‘story problems are just too hard,’ or ‘that nobody can do word problems,’ and skip over the material. (p. 6)

In highlighting the several factors that must be dealt with efficiently in order to solve an AWP, Bernardo (1999) stated,

Word problems are often considered the most challenging problems students have to solve in mathematics education. In solving word problems, students have to assemble concepts and procedures and apply them towards the solution of one problem. Failure to solve the problem can come from an error in just one of the range of concepts and procedures applied. However, the most basic difficulty students have in solving word problems lies in the ability to understand the

mathematical problem structure that is embedded in the problem text. Difficulty with comprehending the problem structure often leads to errors in the choice of the solution strategy. (p. 149)

Current students often mistakenly believe themselves to be the sole sufferers in the long saga of algebra instruction, but historical records indicate that such is not the case. The Rhind Mathematical Papyrus, which according to Katz (1998) was “copied about 1650 B.C.E. by A’h-mose from an original about 200 years earlier,” (p. 3), is an ancient document containing much of the mathematical knowledge and practice of ancient Egyptian civilization. The papyrus “is a practical handbook of mathematical exercises . . . the 85 problems contained herein give us a pretty clear idea of the character of Egyptian mathematics” (Burton, 2003, p. 35). Many significant problems from the papyrus provide insight into the type of problems solved by the ancient Egyptians and were posed in written language. These problems originated centuries prior to the development of algebraic symbolism. The ancients were accustomed to verbal and written forms of mathematical communication. Katz (1998) stated:

Neither the Chinese nor any of the other ancient peoples used the symbolism that today enables such problems to be solved with little effort. All the problems and their solutions were written out in words. Even so, the scribes did not hesitate to present problems with unwieldy solutions, perhaps because they wanted to convince their students that a thorough mastery of the methods would enable even difficult problems to be solved. (p. 17)

Consideration of ancient architectural wonders such as the pyramids of Egypt, Stonehenge, and the Great Wall of China suggest that the engineering skills of builders in

those cultures were greatly enhanced by the nature of the problem-solving skills which they mastered.

Reading Comprehension (Factor 1)

The process of solving an AWP begins as the student reads the problem and attempts to make coherent sense out of the several sentences. Aiken (1971) indicated that “the difficulty of reading word problems, in particular, has received a great deal of attention in the literature. Various instructional approaches, emphasizing analysis of the problem and language arts analogies, have been proposed” (p. 308). Multiple readings of the AWP text are essential to the student’s comprehension of the problem statement and context, as suggested by Cook (2006). Bernardo (1999) found that

students were better at comprehending the problem text when it was written in the student’s first and more proficient language. . . . Moreover, students made more comprehension errors that violate the problem structure, and consequently made more solution errors when the problem was written in the student’s second and less proficient language. (p. 150)

Abedi and Lord (2001) investigated the importance of language in student test performance and concluded, “English language learners scored significantly lower than proficient speakers of English . . . [and] it appears that modifying the linguistic structure in math problems can affect student performance” (p. 231). During interviews conducted prior to the testing session, the majority of the participants preferred the linguistically altered test questions saying, “It seems simpler; you get a clear idea of what they want you to do” (Abedi & Lord, 2001, p. 222). Students often experienced difficulty correctly identifying the meanings of mathematical words and phrases, leading to incorrect representations of the context and incorrect solutions. Yerushalmy (2006) stated, “In

solving contextual problems, construction of a symbolic system for real-world knowledge is a complicated task because ‘knowing’ the actual phenomenon cannot be easily manifested in symbolic language, even when one is familiar with the language” (p. 359). For example, students who grasp the concepts of perimeter, area, distance, etc., as stated in AWP are still likely to be deficient in their ability to correctly express the relationships and conditions within the AWP in symbolic terms. Students spend considerable time in learning to read written non-mathematical language during formal educational years, but very little time is expended in efforts to learn to read mathematical writing. One reason may be that the inherent symbolic nature of mathematical writing is quite alien to the student when compared to the written words of non-mathematical communication. Yerushalmy (2006) suggested, “developing competence by solving real-world problems in function-based algebra means learning to move freely between words, expressions, numbers, and symbols” (p. 358). Vilenius-Tuohimaa, Aunola, and Nurma (2007) investigated “the interplay between mathematical word problem solving skills and reading comprehension” (p. 409). Their results suggested that the two skills were in fact interrelated, and they found “a strong correlation between all math word problem types and reading comprehension types” (Vilenius-Tuohimaa et al., 2007, p. 422).

Identifying Mathematical Operation Clues (Factor 2)

As the student reads the AWP statements, they must locate within the text the words that aid them in identifying the mathematical operations being referenced, whether explicitly stated or implied. The student will need to create one or more equation(s), which quantify the relationships based on the operation clues within the problem. Students also experienced difficulty in the identification of the mathematical operations explicitly stated or implied in the problem, often confusing operational clues. Alibali,

Brown, Kao, Nathan, and Stephens (2009) completed a study of middle school students to determine their conceptual understanding of equations, as presented in story problem form. Students were given two major tasks – to solve story problems and to write a story problem to match given equations. The researchers' findings were a) the students were fairly successful at solving algebraic equations, b) students experienced difficulty when the equation involved two operations (e.g., $63 + n - 13 = 91$), and c) the students most often made errors in interpreting the operation of multiplication. Additional difficulty was encountered when multiplication was combined with a two-operation scenario. Pape (2004) distinguished between *inconsistent language (IL)* and *consistent language (CL)* problems and stated:

In CL problems, the unknown is the subject of the comparison sentence, and the relational term is consistent with the arithmetic operation required to solve the problem (e.g., *n times as many* signifies multiplication). In IL problems, the unknown is the object of the comparison sentence, and the relational term is inconsistent with or the opposite of the operation required (e.g. *n times as many* signifies division). (p. 192)

According to Pape (2004), the following is an example of an IL problem:

Last year the sixth grade sold 125 raffle tickets each day. That is 5 times as many tickets as the fifth grade sold per day. How many tickets did the fifth grade students sell in a day? (p. 189)

This problem is considered an IL problem because the object of the second sentence is the unknown quantity and the relational term is inconsistent with the operation (*5 times as many*) required to solve the problem. The corresponding CL problem may be stated as:

The sixth graders sold 125 tickets each day. The fifth graders sold one-fifth of the 125 tickets the sixth graders sold. How many did the fifth graders sell in one day?
(Pape, 2004, p. 189)

In the CL problem statement, the student should be more likely to correctly identify the correct mathematical operational clues and the relationship between the numbers of tickets sold by each group of students. As an additional example of the difficulty of properly identifying operational clues, Pape (2004) used the following:

CL: Joe runs 6 miles a week. Ken runs 3 times as many miles a week as Joe does.
How many miles does Ken run in 4 weeks?

IL: Joe runs 6 miles a week. He runs $\frac{1}{3}$ as many miles a week as Ken does.
How many miles does Ken run in 4 weeks? (p. 192)

Pape (2004) identified three types of errors associated with problem solutions. A *reversal error* occurred when the opposite operation was used in the problem solution, a *linguistic error* occurred when a computational step indicated in the text was omitted, and a *mathematics error* occurred when the student misunderstood a mathematical relational statement or operation. The correct identification of mathematical operation clues and references to physical objects may aid the student in selecting an appropriate sketch to illustrate the physical and/or mathematical relationships in the problem. For example, the problems COIN and COSTS do not require a *geometric object sketch*; but a simple picture showing two differently sized circles to represent the coins would be useful, as the coins have different values. In problems where reference is given to a specific geometric shape, such as a *rectangle*, the operations may be addition to find the *perimeter* or multiplication to find the *area*. Students in Algebra I and Algebra II courses would have had significant exposure to the basic geometric shapes and operations connected to

finding mathematical values connected to such shapes.

Recognition and Use of Geometry/Sketches (Factor 3)

After multiple readings of the AWP text to more fully understand the problem, the student is faced with the task or option of drawing a sketch, or other visual representative of the AWP statement. The importance of using visualization to aid in understanding and solving the problem has been established in the corresponding research. Coulter (2006) investigated the use of technology to create dynamic modeling tools in order to assist students in their attempts to visualize aspects of algebra: “By making representations on screen that are more visual and interactive than what is found in a traditional textbook, we can help in making the transition to algebraic thinking” (p. 18). Battista (1990) suggested, “Spatial visualization and logical reasoning were important factors in geometry achievement and geometric problem-solving for both males and females” (p. 52). As indicated in Table A2 (Appendix A), three of the 15 AWPs required geometric sketches; and in 11 others, some form of visual image was utilized in the proposed solutions. Rasmussen and Marrongelle (2006) developed the notion of “a device, such as a graph, diagram, equation, or verbal statement that a teacher intentionally uses to connect to student thinking” (p. 389). Given that instruction is often a model of problem-solving behavior, when teachers use visual representations to instruct, the students may be more likely to understand and use visuals to solve AWPs. The student, through experience, should and will develop knowledge of when and what kind of visual representations to use. Griffin and Jitendra (2009) stated, “instructional strategies that researchers have found to be consistently effective for teaching students who experience learning difficulties in mathematics include depicting problems visually and graphically” (p. 188). Pyke (2003) found the reading ability and spatial ability of students, as

indicated by their use of words, symbols, and diagrams to communicate about their ideas, to be key in contributing to students' representational proficiencies and solution of problems. Fennema and Tarte (1985) found that "girls tended to use pictures more often during problem solving than boys did" (p. 184).

Uesada, Manalo, and Ichikawa (2007) investigated the perceptions and daily-learning behaviors of New Zealander and Japanese students related to student use of diagrams in the solution of math word problems; a diagram "was defined as any representation of the problem other than words [on their own], sentences, or numerical formulas. Tables counted as diagrams" (p. 327). AWP and a questionnaire were administered to the students. The AWP were either one-object or two-object problems, with the latter requiring coordination of two values to arrive at a solution. The research results indicated that the New Zealander students a) obtained significantly higher percentages of correct answers on both types of AWP, b) produced significantly higher percentages of high quality diagrams, c) used diagrams more easily and confidently, and d) received more teacher encouragement to use diagrams to solve AWP. Uesada et al. reported, "In contrast, significantly higher percentages of the Japanese students did *not* use diagrams in their problem solving *and* produced incorrect answers" (p. 332). The study emphasized the positive potential of the use of diagrams, i.e., visual representations, to promote correct solutions to AWP. Multiple representations for AWP information and relationships were valuable as the student attempted to assimilate and fully understand the problem. The visual clues provided valuable assistance and alternative forms of interpretation and positively impacted the student's ability to grasp implicitly and explicitly stated relationships within the AWP.

Recognizing Relational Statements (Factor 4)

One of the proposed factors is that students be able to recognize relational statements between components of the AWP. Many mathematical procedures are concerned with expressing the relationships between quantities. An equation is a symbolic statement of the relationship between two or more quantities, and the student creates the appropriate equation/inequality based on an understanding of the relationships stated or implied within the problem. Material presented in algebra textbooks related to the solution of AWP's begins with a list of key words and phrases indicative of the mathematical operations. The simply stated phrases *less than*, *decreased by*, *more than*, *at least*, *at most*, etc., were often misinterpreted by students. Reed and Ettinger (1987) stated, "constructing a correct equation for a word problem requires that students (a) represent correctly the relations between the variables and quantities in the problem and (b) enter these values into a structurally correct equation" (p. 44).

Yerushalmy (2006) stated the difficulties encountered by students in attempting to solve an age problem: "They had difficulty expressing the story in a correct algebraic model" (p. 373), and their attempts in "constructing an equation required seeing the relations between the processes and describing them algebraically" (p. 375). In several types of AWP's, quantities within the problem simultaneously changed--i.e., objects moving simultaneously or persons aging together--but students often failed to apply the process of change equally to all affected quantities when writing equations. In his study of students in seventh through 9th grade algebra, Yerushalmy (2006) observed that many students were in the 9th grade before they gave "serious consideration to the meaning of equations, variables, and expressions of functions" (p. 383).

Students in pre-algebra, post-algebra, and at-risk situations also encountered

similar challenges in learning to use algebraic methods. In a study of second through fourth graders undertaken over a 30-month period during which the students were given instruction in basic use of variables, Carraher, Schielmann, Brizuela and Earnest (2006) found that the students were able to develop moderate facility with the use of variables. The students learned to use a variable to represent an unknown quantity in a simple word problem statement and addition and/or subtraction to solve the problem. They “understood the relations between the daily amounts of each protagonist in the story problem; they also understood how the amounts on each day related to the starting amount” (Carraher et al., 2006, p. 107).

In a study undertaken with adult teachers in algebra courses taken for advanced credit, Graham and Honey (2009) identified and commented on two points regarding student abilities: “To a greater or lesser extent, they were able to cope with simple manipulative skills like collecting like terms and solving simple equations, [but] what they all struggled with was actually setting up the problem algebraically in the first place” (p. 212). There was no specific mention of whether this was related to their failure to recognize relational statements between key parts of the AWP or to write the necessary equation or inequality prescribed by the constraints of the problem. Xin and Zhang (2009) explored the effects of teaching *conceptual model-based problem solving* (COMPS) to students at-risk for mathematics disabilities. Their method included instructional intervention designed to teach the students how to follow a four-step procedure to solve mathematical word problems. Xin and Zhang (2009) asserted, “First, detect the problem type . . . Second, organize the information in the conceptual model diagram. Third, transform the diagram into a meaningful mathematical equation. Fourth, solve for the unknown quantity or variable and check the answer” (p. 432). These four

objectives were very similar to factors suggested in the current research: a) reading comprehension, b) identifying operational clues, c) writing equations, and d) solving the problem and checking the answer. The results of the Xin and Zhang's (2009) research indicated that the students did significantly benefit from the specialized instruction, as seen in their improved problem-solving ability.

Moseley and Brenner (2008) investigated the effect of two different instructional methods, *standards-based approach* (SBA) and *traditional instruction* (TI), on students' performance in word problem representation and solution skills. Comments made within their research connect to several aspects of the current study. They divided the problem-solving process into two main components: *problem representation*, consisting of both translating the problem into a mental representation and then integrating that mental representation to form a formal mathematical structure such as an equation, expression or formula, and *problem solution*, consisting of both devising a strategy to work with the formal mathematical structure created in the representation phase and then carefully monitoring each step as progress is made to completion. Accordingly, the SBA method emphasized the student-centered creation of diverse ways of representing the problem, whereas the TI method encouraged the use of rote procedures and skills. In both methods, the students were asked to identify relational statements within the problem and use variables and operations to create equations. To highlight the prevalent difficulties experienced by students at various levels in writing relational statements, Moseley and Brenner (2008) mentioned, "37% of college engineering students could not provide a correct equation for the relational statement 'There are six times as many students as professors at this university' " (p. 6). Yerushalmy (1997) wrote, "Algebra is viewed as the study of relationships between quantities" (p. 432). The importance of recognizing

relational statements and using sketches lies in the student being able to connect both of these to a known formula to compose an equation that is then solved to arrive at an answer.

Formula Selection (Factor 5)

Formulas are equations expressing relationships between quantities, such as *the perimeter of a rectangle is equal to two times the sum of the length and the width*, for example, the formula $P = 2(L+W)$. The equation created is typically based on a formula appropriate for the AWP statement, such as perimeter, area, volume, distance, concentration, or interest calculation, etc. The use of incorrect formulas leads to erroneous solutions. The formula choice is typically a matter of previously learned knowledge. Nathan, Kintsch, and Young (1992) suggested, “An inability to access relevant long-term knowledge [can] lead to serious errors” (p. 1). Proper formula selection leads the student to write an equation, based on the formula structure, in which specific values within the AWP replace variables within the formula. The formula is a pattern used to symbolically state the relationships expressed in the AWP.

Translation of Text and Equation Writing (Factor 6)

At the core of solving AWP is the process of using manipulative algebraic skills to solve one or more equations. The student, based on a cumulative understanding of the problem context, relationships, operations, etc., creates the equations. Swafford and Langrall (2000) worked with sixth graders, investigating their ability to create and use equations to solve six problems of linear and non-linear form, and found that “all but one were able to generate an equation for at least one of the situations” (p. 101). The students used simple linear equations including variables to represent the relationships within the problem statements. In a similar study, Graham and Honey (2009) found that all students

struggled with actually setting up the problem algebraically in the first place. Moseley and Brenner (2008) indicated that students must be “able to successfully write algebraic notation so that it models the mathematical relations presented in pictorial [or written] form in a problem . . . and participants must be able to convey the problem representation as a mathematical statement that has at its core a variable that is being altered by arithmetic operations” (p. 7). Yerushalmy (2006) investigated the use of computer software by slower algebra students to solve a variety of AWP. The full utilization of the software required the students to form “an abstraction of the morphism between terms of the real world and formal algebraic symbols to create a symbolic model” (Yerushalmy, 2006, p. 361), a function or an equation. Yerushalmy (2006) noted that “unless they dealt with a problem of a type with which they had relatively long experience, they delayed using formal symbolism” (p. 382), opting instead to use numeric or graphical solution strategies. This suggested that the ability of students in writing equations might be related to their overall experience in solving AWP.

Equation Solving (Factor 7)

Once the student has created the correct equation based on an appropriate formula, then the student must apply previously learned rote-manipulative procedures to obtain the solution of the equation. This typically is the point at which the student’s likelihood for overall success greatly improves, as most students will be proficient at solving equations. The inherent danger is that the student will not correctly remember or will misapply the various manipulative steps. Graham and Honey (2009) indicated, “To a greater or lesser extent, students were able to cope with simple manipulative skills like collecting like-terms and solving simple equations” (p. 211). Webb, Gold, Qi, and Novak (1990) found, “Students can memorize algorithms for clearly identified problem types

presented in conventional ways, and yet be unable to solve problems involving the same concepts but presented in symbolic forms” (p. 5). Nathan and Koedinger (2000) investigated the “relationship between teachers’ and researchers’ predictions about the development of algebraic reasoning and students’ performances” (p. 168). In the research, teachers and researchers were asked to rank mathematics problems, which had been administered during 1998 and 2000 to two groups of students, according to the perceived difficulty that students would have in solving the problems. The three categories of problems were a) symbolic equations, b) word equations, and c) story problems. The teachers and researchers predicted that the students would have the most difficulty in solving the story problem and the least difficulty in solving the symbolic equation, but student performance was opposite to the predictions. The students did employ traditional algebraic methods of undoing operations to solve equations. A purported reason to explain the students’ ability to solve story problems more efficiently than symbolic equations lies within the framework of the educational development of the students; they had dealt with verbal and written language reasoning for their entire educational experience but had recently begun to work within the framework of algebraic symbolism and procedures. The proposed answer was derived from, and must satisfy the conditions of, the original problem statement. Too little instructional emphasis is given to the importance of the connections between the final two steps of solving problems—*finding* and *checking* the answer. Far too often students believe any answer that they get to a difficult problem must be correct, and they fail to check the answer against the original problem conditions. This is a reason to teach critical thinking skills more effectively.

Checking Solutions (Factor 8)

The student who can nimbly solve the equation often seems to fail in the ability to consistently and properly diagnose the existence of an incorrect answer. The problem may have constraints, such as non-negative or integer solutions, which the student does not recall when declaring a solution. This unfortunate occurrence does have a potential long-term remedy: additional experience in solving similar AWP. According to Weaver and Kintsch (1992), “Students’ ability to solve these problems can be improved, of course. Performance is often enhanced by prior exposure to similar problems or by using a worked-out-example as a guide” (p. 419), and “How well students can use what they remember from earlier problem solving depends on their skill in mapping this new knowledge to the new problem” (p. 427).

Additional Factors

The final two factors of potential importance are student mathematical learning style and gender. Keast (1998) noticed that teachers changed their teaching styles based on whether the students were boys or girls, or in single-sex classrooms; “The girls formed small learning groups based on the tables where they sat. Their learning was a sharing process with lots of discussion and developing of ideas in a connected way. . . . Boys in the single-sex class disliked group work and it was very difficult to get boys involved in discussions of their understanding of mathematics” (p. 56).

Alternative or additional forms of presentation of similar material can enhance the students’ understanding and comprehension of the problem. A picture drawn to accompany the AWP text or a homemade machine to illustrate complex and connected motions may strike a resonating chord with some non-auditory learners. Much comment has been made over many years of the preconception that males are better at math and

females are better at language arts. Battista (1990) suggested, “Whereas males and females differed in spatial visualization and in their performance on high school geometry, they did not differ in logical reasoning ability or in their use of geometric problem solving strategies” (p. 47). Many AWP’s require a logical reasoning ability to comprehend relationships and changes between variables. Fennema and Carpenter (1981) reported that although females score higher at numeration skills at ages 9 and 13, they typically fall behind males in performance in higher-level mathematics classes such as Algebra I, Geometry, and beyond Algebra II, when measured at the age of 17. Halpern’s research (as cited in Rathus, 2010) reports that as a group, females surpass males in verbal ability throughout their lives, which may account for any discernable gender differences in ability to correctly read and comprehend the AWP text. Hyde, et al. (as cited in Rathus, 2010) found that boys began to outperform girls in word problem solution in high school and college.

In summary, students must possess and employ a diverse selection of mathematical skills, as identified by the eight factors, in order to successfully solve the variety of AWP’s encountered in high school algebra and post-algebra classes.

Chapter 2: Literature Review

Overview

Webb et al. (1990) stated: “Researchers in mathematics and mathematics education and cognitive psychologists have long recognized that a very important, if not essential, component of successful problem solving is the ability to translate between different symbolic representations of information” (p. 1). Of the eight factors proposed by the researcher, factors 2, 4, and 6 all required some form of translation from text to mathematical symbolism. According to Mayer, in the work edited by Sternberg (1985), “Mathematical [word] problem solving can be broken down into two major parts: *problem representation*, converting a problem from words into an internal representation; and *problem solution*, applying the legal operators of mathematics to the internal representation in order to arrive at a final answer” (p. 131). Hegarty et al. (1995) stated, “Unfortunately, students perform particularly poorly on arithmetic word problems even when they perform well on corresponding arithmetic computation, suggesting that problem comprehension is a source of students’ difficulties” (p. 76).

Research Questions

Based on agreement with the preceding evidence, the current research questions were related to the problem representation aspect of AWP solution. Three specific questions provided a focus for the research. These are listed without regard to potential order of importance.

1. To what degree does the student’s ability to identify written clues indicative of mathematical operations impact AWP solution performance?
2. To what degree does the student’s ability to recognize relational statements between component parts of the written text impact AWP solution performance?

3. To what degree does the student's ability to translate written text into mathematical equations impact AWP solution performance?

The following review of the literature focused on each of the three questions in turn.

Identifying Mathematical Operation Clues

The first research question related to an aspect of mathematical problem representation, specifically *translation*, as described by Mayer, in the work edited by Sternberg (1985). The student was required to create a representation of the AWP context in order to solve the problem, and “representation . . . involves translating each sentence from English into some other form, such as an equation” (Sternberg, 1985, p. 130). Mayer further proposed, “In order to translate each proposition in a story or word problem, a problem solver needs some knowledge of language (*linguistic knowledge*) and some knowledge about the world (*factual knowledge*). Linguistic knowledge is required to parse the sentence into variables” (Sternberg, 1985, p. 132). The typical algebra textbook contains a section of instruction within which the student studies words and/or phrases used to indicate various mathematical operations within verbal statements or written text. The following is a sample of common words and phrases used to indicate the basic mathematical operations of addition, subtraction, multiplication, and division, as taken from an algebra textbook: “*more than, less than, greater than, added to, subtracted from, the sum of, twice, three times, multiplied by, the product of, divided among, double, half of, fewer than, increased by, difference of, ratio of, quotient of*” (Smith, et al., 1990, p. 30). Alibali et al. (2009) found that “students have substantial difficulty generating stories to correspond with algebraic equations” (p. 4). An analysis of the typical errors made by the students indicated a weakness or incompleteness in “their conceptual understanding of some arithmetic operations- in particular, multiplication” (Alibali et al.,

2009, p. 4). The findings of Alibali et al. (2009) called “for an increased focus on operation sense . . . the meanings of the arithmetic operations should be an explicit focus of instruction in early grades . . . and students could benefit from instructional activities that focus on combining multiple mathematical relationships” (p. 14). Sowder (1988) documented middle school students’ difficulties in identifying which operations need to be performed to solve story problems. Dixon, Deets, and Bangert (2001) reported that students’ intuitive understandings of multiplication are weaker than their understandings of addition. According to Yershalmy (1997), students had difficulties solving and symbolizing story problems that involved multiple operations.

Articles and research have been mentioned that address the reading-comprehension concerns for the student. Acosto-Tello (2010) conducted a study in which comparisons were made between the readability grade level of the AWP and the resulting student proficiencies in reading and math as indicated on state assessment reports. Results indicated that the students “progressively performed worse in mathematics as they moved through the grades” (Acosto-Tello, 2010, p. 22). Current efforts to effectively integrate reading (and writing) practice into the mathematics curriculum further support the need to improve the teaching of AWP solution procedures. Ilany and Margolin (2010) stated that the “difficulty with the solution of mathematical word problems is the need to translate the event described in natural language to arithmetic operations expressed in mathematical language. The translation from natural language includes syntactic, semantic, and pragmatic understanding of the discourse” (p. 139). The difficulty of translation highlighted the necessity that the student be able to identify the textual clues suggesting specific mathematical operations and be able to understand the “literal clues, that is the words that support (*helpful clues*) or the words

that deceive (*misleading clues*), as clues for choosing the arithmetic operations needed to solve the problem” (Ilany & Margolin, 2010, p. 143). Correct identification of the mathematical operations in the AWP text is essential to the subsequent creation of the equation(s) to be solved in the complete AWP process.

Recognizing Relational Statements

Polya (1957) indicated that understanding the problem is the first step a student must take in the solution of a problem. Understanding the problem required identification and comprehension of explicit and/or implicit relationships within the problem text. Ilany and Margolin (2010) suggested that the student is involved in “defining the problem and comprehending the situation it describes; building a mathematical model of the mathematical principles relevant to the problem; understanding the relationships and the conditions pertaining to the problem” (p. 143). Furthermore, in emphasizing the importance of word problems,

to read a word problem in mathematics and give it meaning, it is necessary to perceive the problem as a textual unit and not as a collection of data. ... Indeed, exercises like addition and subtraction or multiplication and division are important for the understanding of the mathematical language, but the perception of the textual structure is a process by which you can identify textual components and carry out different logical operations. (Ilany & Margolin, 2010, p. 138)

Ilany and Margolin (2010) concluded that “graduated work on solution methods of word problems using schemas built in previous work on word problems will enable students to cope with more complex problems . . . and necessitates the implementation of a number of cognitive actions” (p. 147).

The appropriate equation necessary to solve the AWP must accurately represent

the relationships expressed within the AWP. Clement (1982) suggested “that schools have been more successful in teaching students to manipulate equations than they have in teaching students to formulate equations in a meaningful way” (p. 29.) In order to solve an AWP, the student must create an appropriate equation that correctly matches the relationships expressed in the problem; therefore, identifying the relationships precedes creation of the equation(s). In an analysis of various factors affecting problem difficulty, Loftus and Suppes (1972) “found that the hardest problem in their set was one that contained relational propositions: ‘Mary is twice as old as Betty was two years ago. Mary is 40 years old. How old is Betty?’ ” (p. 132). Mayer, in the work edited by Sternberg (1985), identified relational propositions as expressing a quantitative relation between variables, and further stated, “There is ample evidence that students have difficulty in representing relational propositions” (p. 132). Low, Over, Doolan, and Michell (1994) stated that an understanding of the structure of the problem is essential:

In the following river current problem: *If the boat traveled downstream in 2 hours with a current of 8 km an hour and the return trip against the same current took 3 hours, what would have been the speed of the boat in still water?* A student must know that the problem involves specific relationships between speed of boat, rate of current, and time in order to determine what information from the text should be used, in what sequence, and through what operations. (p. 424)

In a study involving 208 11th graders, Low et al. (1994) determined that intervention training in *text editing* facilitates solution of AWP. Within text editing training, the student learned to identify the relevant information necessary to solve an AWP. The possibilities were that the problem had *necessary and sufficient* information, possessed *irrelevant* information, or had *missing and essential* information. Low et al. (1994) also

stated:

Text editing provides a relatively direct measure of schematic knowledge, or understanding of problem structure, because students can detect what information is irrelevant or missing within the text of a problem, only if they know exactly what information is necessary and sufficient for solution of the problem. (p. 425)

In addition to identifying essential information within the AWP context, the student was faced with the task of determining the meanings of the multiple sentences. Wollman (1983) found that in “about 70% of all errors, [the student] failed to extract an adequate meaning from the sentence either before or after writing an equation; for example, they did not determine which quantity was the greater” (p. 179). This represented a failure to properly identify the explicitly stated or implied relationship between two or more quantities within the AWP statement. Hall, Kibler, Wenger, and Truxaw (1989), in a study of 85 undergraduate computer science majors at their junior or senior level, investigated the extent to which problem solvers were able “to generate a solution-enabling representation of a problem . . . and able to assemble quantitative constraints under the guidance of their understanding of the *situational context* presented by the story problem” (p. 226). In their rationale, these students “could be viewed as ‘experts’ in algebra story problem solving, because they must have successfully completed courses in algebra during secondary school” (Hall et al., 1989, p. 242). The students were presented with four algebra story problems and given a short period of time in which to solve each, supporting their solution with complete notes and work, after which they were given additional time to explain their solutions. The student solutions were examined for various types of errors. The two primary error types were *conceptual errors* and *manipulative errors*. Conceptual errors occurred “when a student either includes a

constraint that is inappropriate for the problem or excludes a constraint that is a critical requirement” (Hall et al., 1989, p. 253). These were referred to as *errors of commission* (i.e., incorrect operations used based on misunderstanding the context) or *errors of omission* (i.e., ignoring an explicitly stated relationship in the problem statement), respectively. The second type of primary error was a manipulative error, further identified as an *algebraic error*, *variable error*, or an *arithmetic error*. For variable errors, there were two sub-types: *switch errors* (i.e., the meaning of the variable was changed in the course of problem solving) and *label errors* (i.e., variables were used as labels for quantities rather than as unknown values to be determined). Arithmetic errors were miscalculations.

During the problem solving sessions, the students created models of algebra story solving, typically of the generative nature. A model with “generative capacity uses expressive language for describing problems and their solutions to produce descriptions of problem-solving activity that obey certain constraints” (Hall et al., 1989, p. 225). The student was required to create one or more equations that correctly expressed the relationships and were subject to the constraints stated or implied within the story problem and were subsequently used to solve the problem. Hall et al. (1989) suggested that algebra story problems have:

two levels of abstraction: the *quantitative structure* of related mathematical entities and the *situational structure* of related physical entities within a problem. . . . By the quantitative structure of algebra story problems, we mean the mathematical entities and relationships presented or implied in the problem text. (p. 227)

The second and third research goals of the current study were focused on determining the

student's ability to correctly identify and express these relationships in equation form suitable for problem solution. The error analyses of the Hall et al. (1989) study found:

Conceptual errors of omission and commission increase for the more difficult problems . . . and appear much more frequently than manipulative errors (arithmetic, algebraic, or variable errors) on all problems, furthermore, manipulative errors within algebraic and arithmetic formalisms do occur, but these are overshadowed by conceptual errors of omission or commission as a primary source of problem-solving difficulty. (p. 259)

According to Hall et al. (1989), "Thus, a substantial portion of a problem solver's activity is devoted to reaching an understanding of the problem that is sufficient for applying the routine of formal manipulation" (p. 269). The student's attempts to solve an AWP must include correct identification of explicit or implied relationships within the problem text in order to write the equation(s) to be solved in the complete AWP process.

Translation of Text and Equation Writing

The third research question related to the student's ability to create an equation, based on the AWP context and relationships, which would then be solved. This required a translation from the English language text into mathematical symbolism. Polya (1957) suggested that for the problem to be understood, it is first of all necessary for the verbal version to be understood. Students in an Algebra II course would have spent at least 9 years studying the English language but perhaps only 1 or 2 years in mathematics courses where the emphasis would be on learning the specifics of mathematical language. In the solution of AWP, students encounter both natural and mathematical language. Kane (1970) clearly identified a source of complexity and difficulty that students encountered as they attempted to solve AWP: "In the solution of word problems, that is to say in the

solution of mathematical problems that are accompanied by text, the student is faced with two languages mixed together: natural language and mathematical language” (p. 580).

Mathews (1997) stated, “The most difficult aspect of solving an AWP seems to be translating from the written representations to the symbolic algebraic representations” (p. 131). Clement, Lockhead, and Monk (1981) suggested, “The process of translation between a practical situation and mathematical notation presents the student with a fresh difficulty that must be overcome if the application (or the mathematics) is to make any sense to the student in the long run” (p. 287). Furthermore, Ilany and Margolin (2010) added, “To read a word problem in mathematics and give it meaning, it is necessary to perceive the problem as a textual unit and not as a collection of data” (p. 138.) It was common for students to attempt to determine the several bits of information relevant to the problem solution rather than see the problem from a global perspective. Ilany and Margolin (2010) pointed to underlying causes for the significant difficulty of students in solving AWP:

Mathematical language is a language of symbols, concepts, definitions, and theorems. Mathematical language needs to be learned and does not develop naturally like a child’s natural language. In mathematical language the child learns to recognize, for example, numbers as objects, one to one of their similar and different properties. The child perceives numbers as signs of which it is possible to perform calculations and to do various manipulations. (p. 138)

Ilany and Margolin’s (2010) central argument can be found in the following:

There is a bridge between mathematical language that necessitates seeing the mathematical components, and natural language that demands textual literacy for the text as a whole. In other words, there is a bridge between the mathematical

components and the literal components. (p. 138)

Unfortunately for the student, the “structure of mathematical language is more precise and less flexible than the structure of natural language, thus great tension is created in the use of natural language in mathematical problems” (Ilany & Margolin, 2010, p. 139). An essential difference between natural language and mathematical language was that “in natural language the order of the words determines the meaning” (Ilany & Margolin, 2010, p. 140). Ilany and Margolin (2010) further concluded that the

process of dealing with the verbal text of the mathematical problem is multi-staged, and necessitates the implementation of a number of cognitive actions: interpreting symbols and graphs, understanding the substance, understanding the linguistic situation, finding a mathematical model, and matching between the linguistic situation and the appropriate mathematical model. (p. 147)

According to Wollman (1983), “the ability to translate sentences into algebraic relationships figures heavily among the problem-solving skills required in quantitative science courses and mathematics courses in secondary school and college” (p. 169). The most common error, called a *reversal*, seems “due to an attempt to make the sequence of algebraic symbols match either the word order of the sentence (called a *word order match*) or a non-algebraic graphic representation of the meaning of the sentence (called a *static comparison* or *passive semantic approach*)” (Wollman, 1983, p. 170). Clement (1982) asked college students to write equations to represent a proposition such as, “There are six times as many students as professors at this university” (p. 17). Nearly one-third of the students produced an incorrect equation, such as $6S=P$. The difficulty was connected to two approaches: the *static approach* or *syntactic translation*, in which the student overemphasized literal translation of the words, and the *procedural approach*

or *semantic translation* in which the words and text are interpreted as procedural instructions about how to convert one variable into another. Wollman (1983) found that “the success rate on the *students-professors* sentence was unaffected by common contextual knowledge, namely, that as a rule students outnumber professors . . . suggesting that students do not profit from their contextual knowledge in the translation process--that they fail to use even an obvious bit of relevant knowledge” (p. 171). Bernardo and Okagaki (1994) commented on this same problem, stating that “when they [students] translate word problems into equations . . . as they order the information in the constructed equation, they simply follow the syntax of the [written] statements” (p. 212). The students “ignore both the semantic meaning of the sentence and the symbolic meaning of the equation.” Additionally, Bernardo suggested that the “individual must see the equation as a dynamic representation of an operation, rather than as a static representation of verbal information.” (Bernardo & Okagaki, 1994, p. 212). Clement (1982) gave another problem to college students: “At the last company cocktail party, for every six people who drank hard liquor, there were eleven people who drank beer” (p. 17). Fifty-five percent of students translating the statement into an equation did so incorrectly.

According to Ilany and Margolin (2010), errors in translation from the natural to the mathematical language frequently occurred due to the students’ lack of command of the language, i.e., their inability “to construct a meaningful body of knowledge from the information in the question, including data and a solution scheme” (p. 140). In other research, Clement (1982) referred to the correct translation strategy as “the *operative approach*, an equation-writing strategy involving the tacit assumptions and meanings underlying our conventions for algebraic notation.” (p. 21). Bernardo and Okagaki

(1994) suggested two reasons for the failure of problem solvers to use the operative approach: a) individuals misunderstand “the assumptions and meanings of the mathematical symbols that are used in the equations” and b) “individuals fail to use their knowledge about mathematical symbols because the problem context does not provide enough cues to make the symbolic knowledge readily accessible” (p. 213). Bernardo and Okagaki (1994) concluded that providing training in *symbolic knowledge*, in the form of reminders about the proper use of mathematical symbols, improved performance on equation-writing tasks (p. 217). In contrast, if the word problem context was significantly different from the context within which the symbolic knowledge was typically used, then the learner was less likely to access the relevant symbolic knowledge in order to write a correct equation (Bernardo & Okagaki, 1994, p. 218). The difficulty inherent in the *translation* process appeared to cross ability levels. In a study conducted by Clements (1980), in one group it was “found that 27% of the errors committed by seventh graders occurred during the transformation stage (i.e. translating the variables into equations) . . . and within the group of *low-achieving and average* seventh graders, 28% of their errors occurred during the transformation stage” (p. 126).

Several types of errors were commonly made during the translation process, as stated in prior studies. In a study involving 84 university freshman-level Intermediate Algebra class students, Travis (1981) conducted an error analysis in the student attempts to correctly write an equation matching an AWP statement. Ten problems were presented to each student with the instruction “to read each problem and write an algebraic equation reflecting the given information for the problem” without any attempt to solve the AWP (Travis, 1981, p. 3). The student equations were compared to correct equation formats for each problem statement, and the various incorrect equation forms

were analyzed for various types of errors. The study focused on a *direct translation* procedure, wherein the [AWP] statement is transformed into a mathematically equivalent equation. Travis (1981) stated the following:

The transformation rules alter the current status of the problem and identify the operations relevant to the solution. Some of these are mandatory substitutions (such as 2 *times* for *twice*), function tags (indicating the grammatical function of the word), identification of conventional and relational naming of the variables, and the use of auxiliary representations and cues. Conventional naming would be exemplified by the use of a new variable to identify an additional unknown, whereas relational naming would connect or relate the information using a single variable. Auxiliary cues use a conventional knowledge structure called *frames* that help account for the ability to recognize and interpret the relevant information in a setting. Cues can provide the trigger for processing the heuristics needed to classify problems and retrieve from memory useful information. A direct translation scheme has to be augmented by specific semantic knowledge to insure full understanding of the problem. (p. 2)

Errors identified within the study were varied from problem to problem, but there were several consistently repeated errors. Relational naming errors (i.e., three more than twice a number), function tag misinterpretation (i.e., three consecutive integers), relational naming attached to the wrong variable, violating the Piagetian principle of conservation, failure to correctly differentiate between nature of information presented (i.e., *value* of coins contrasted with *number* of coins), failure to recall auxiliary information (i.e., $d = r \times t$), and misuse of conventional variable procedures (i.e., $L = \text{length}$, and $W = \text{width}$). In several problems, as many as 35 or even 50 errors were identified within the students'

equations. Noticeably fewer operation identification errors occurred than any other type. Travis (1981) suggested, “Word problem solution is dependent upon several types of knowledge, most important is conceptual knowledge for problem representation that leads to the appropriate selection of action schemata for solution” (p. 46). Travis (1981) also suggested that the successful translation phase of the problem statement into the appropriate equation depends upon the student’s conceptual knowledge of various types of AWP’s and the mathematical formulas which best relate to the situation and relationships expressed within the AWP’s. Travis (1981) further suggested, “Older students are more successful in solving word problems because of the availability of appropriate schemata for problem representation and the ability to utilize such schemata to differentiate instances of a problem and determine the nature of the unknown” (p. 46).

In a study of 10 Algebra II students in efforts to complete a series of five AWP’s, Lumpkin and McCoy (2007) identified that “incorrect responses could be linked to an error in one of three places along the problem solving process: an incorrect interpretation of or inability to understand the problem, flaws in the setup of the problem, and/or errors in computation” (p. 99). It was further proposed “that the difficulties students have with word problems, arise from both the differences in student cognition as well as with the structure and context of the word problems themselves . . . as evidenced by an improper interpretation of the problem” (Lumpkin & McCoy, 2007, p. 99). Students who did display understanding of the problem were not always able to set up the problem correctly. In addition, “students that did not understand what the problem was asking were unable to properly set up the problem in a way that could provide a meaningful answer” (Lumpkin & McCoy, 2007, p. 100). The students seemed frequently unable to fully understand the situational context of the problem, leading to incorrect

representations. Lumpkin and McCoy (2007) further stated that “it cannot be assumed that the ability to solve problems will stem naturally from learning the mathematics necessary to solve it . . . students struggle with translating the story of the problem into the symbolic language of mathematics” (p. 101).

The challenging task of translation may be made more difficult with the increasing complexity of the AWP. Mathews (1997), in a study of student performance in solving AWP, used a variety of test questions and procedures, which differed in the number of equations, the number of unknowns, and the number of variables utilized to solve the problems. Students in Algebra I courses generally encounter AWP that limit the number of variables identifying the unknowns to one. AWP found in Algebra II courses will most likely be solved by the use of two equations and two unknowns. In the study, Mathews (1997) found the following:

Student’s worst difficulties seem to arise from trying to determine which single unknown should be defined as the one allowed variable [in one variable solution procedures] and how to translate all of the other unknowns in their heads while at the same time writing some single equation to represent the problem. (p. 123)

Two unknown solution procedures were found to be easier, but there still remained the challenge of properly translating the other remaining variables and creating the proper equation based on the relationships expressed in the problem. Consistent with Mathews’ (1997) findings, “Their ability to translate prose into algebraic symbols, and hence to solve the mathematics problems, increases dramatically” with the use of two unknowns (p. 123). Accordingly, Mathews (1997) suggested that the use of two unknowns is associated with a decrease in the difficulty in translation of problem text into algebraic symbolism and equations for problem solution.

Webb et al. (1990) stated,

Problem solving often involves translating from the symbolic representations of the problem given (typically words and numbers) to another symbolic form that more readily leads to a solution (e.g., diagram, graph, picture, algebra, words, or some combination of these.) . . . and students at all ages have difficulty translating from one representation to another. (p. 1)

In summary, the translation process involves identifying mathematical operations, recognizing explicit and implied relationships, and translating AWP text into solvable equations.

Chapter 3: Methodology

Introduction

According to Xin and Zhang (2009), “Current levels of elementary and secondary students’ mathematics performance suggest that the United States is not preparing the general population with the levels of mathematics knowledge necessary for the 21st century workplace” (p. 427). As stated within a report by the National Research Council (2001), “Assessments conducted at state, national, and international levels over the past 30 years have indicated that U.S. students are notably deficient in their ability to solve mathematical problems” (p. 4). In an effort to assess high school Algebra I, Algebra II, and Pre-Calculus students’ problem-solving abilities, the current study investigated and measured various student performance factors related to AWP solution ability. More specifically, measures of student ability to identify mathematical operation clues, to recognize relational statements, and to translate text into algebraic symbolism were determined. The assessment instruments provided for the collection of quantitative and qualitative data, leading to a mixed-model research design. Measures of rank-order correlation and measures of association were determined from various analyses of the data collected. One-way and two-way ANOVA procedures were also used to analyze the data.

Participant Characteristics and Sampling Procedures

Participants included 163 students (92 females [56.4%], 71 males [43.6%]) from nine classrooms (four Algebra I, three Algebra II, and two Pre-Calculus) taught by five teachers in three high schools in a town in the southeastern United States. Regarding participants’ ethnicity, 46 (28.2%) were African American, 104 (63.8%) were Caucasian, and 13 (8.0%) were Hispanic. Regarding participants’ grade levels, 102 (62.5%) were

9th graders, 19 (11.7%) were 10th graders, and 42 (25.8%) were 11th graders. Regarding participants' course enrollment, 102 (62.5%) were Algebra I students, 43 (26.5%) were Algebra II students, and 18 (11.0%) were Pre-Calculus students. The Algebra I classrooms were comprised exclusively of 9th grade students. There was only one course repeater in the participant group, a student in Algebra II. Frequency values for course and grade composition, by gender and ethnicity, are provided in Figure 1.

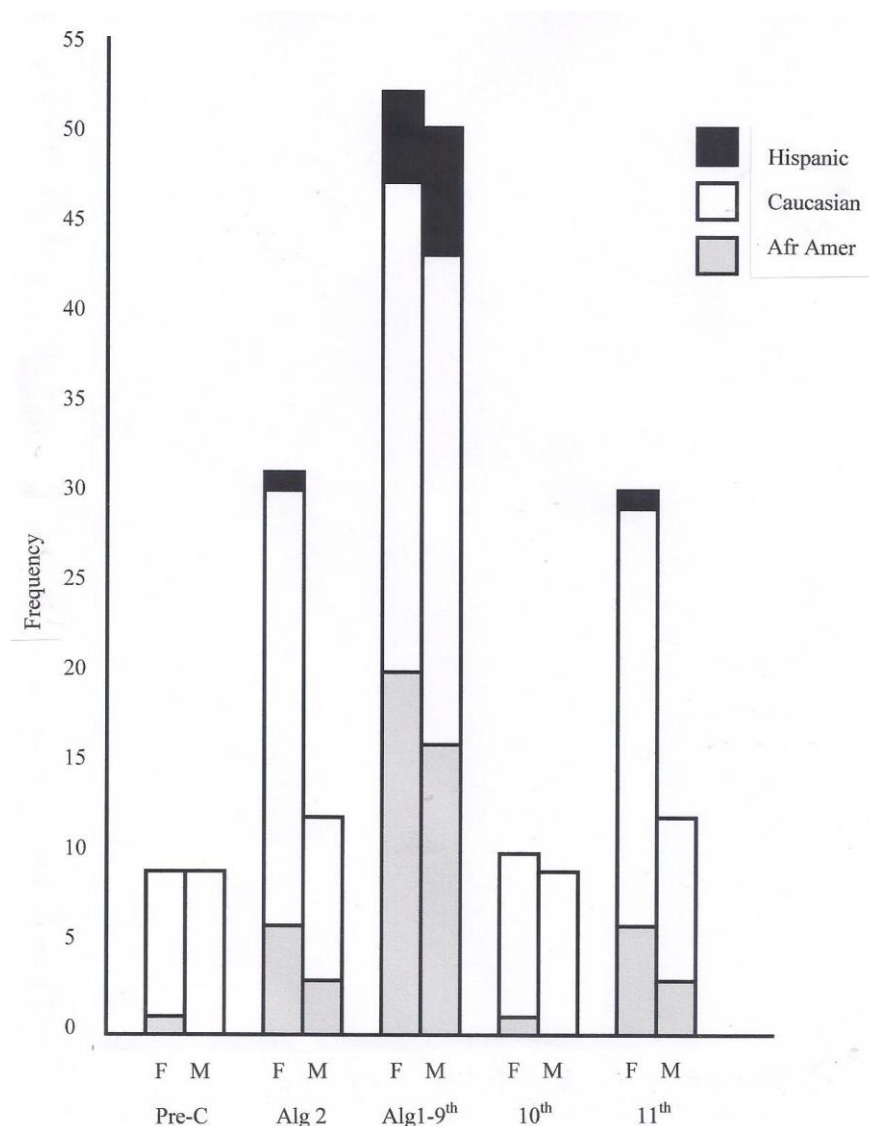


Figure 1. Course and Grade Composition Frequency, by Gender and Ethnicity

The research participant population consisted of those students for whom Parental

Consent and Student Assent Forms were completed and returned. Reasonable effort was taken to identify ESL or multi-lingual learners and six (3.7%) of the participants identified themselves as ESL learners. Due to the significance of reading comprehension ability to overall AWP completion, data from participants with special education accommodations for reading assistance were not included in the research analyses. Cluster sampling of entire Algebra I, Algebra II, or Pre-Calculus classes within the individual high schools were utilized to obtain the complete participant group. The high schools selected were based on the current semester offering of at least one section each of Algebra I and Algebra II courses, and possibly Pre-Calculus. Permission to conduct the research in the high schools was granted by the district and school administrative leadership personnel.

Instrumentation

The data collection instruments are found in Appendix C (Forms A-F). Specific research participant information, of both qualitative and quantitative nature, was collected through a) participant data as supplied by the classroom teacher and recorded on Form A, School and Student Data Record Form for Research Participants; b) Form B, Student Mathematics Learning-Style Questionnaire; c) participant responses to researcher-prepared assessment items related to each of the three research goals, as recorded on Form C, Mathematical Operation Identification; Form D, Recognizing Relational Statements; and Form E, Translating Written Text into Mathematical Sentences; and d) participant solution efforts on a random sample of AWP as recorded on Form F, Student AWP Solutions.

Form A was completed by the classroom teacher and used to gather school demographic data (name, location, student population), course (Algebra I, Algebra II,

Pre-Calculus) and teacher data (years of experience teaching), and participant academic data. The teacher collected data on participant gender, ethnicity, grade level, course repeater status, and ESL status. The teacher assigned a participant code number to participating students. Records were compiled for return of Parental Consent Forms. Quantitative and qualitative data were generated from the forms.

Form B was used to gather data on participant learning styles for mathematics. Responses on the mathematics learning-style survey questions provided information on the participant's prior experiences with solving AWP's and with solving standard equations as well as their learning style preferences in mathematics courses. The data collected was nominal, ordinal, and ratio-level.

Form C was used to gather data on the participant's ability to identify the specific mathematical operation(s) referenced in AWP text. For this instrument, the participant read seven AWP statements. For each of the AWP's, the student underlined or circled the words or phrases within the AWP statement that indicated mathematical operations and then wrote the corresponding operation symbol next to the words or phrases. There were 24 correct responses, with total and percent correct values generated with this instrument. The total number of correct responses, out of 24 possible, was denoted as OA.

Form D was used to gather data on the participant's ability to identify relational statements between component parts of the AWP text. Seven multiple-choice questions were answered. Total and percent correct values were generated with this instrument. The total number of correct responses, out of seven, was denoted as RA.

Form E was used to gather data on the participant's ability to translate the written text of an AWP into appropriate mathematical symbolism and equations. The participant answered eight questions, each requiring the reading of a written sentence and the

selection of the correct equation to match the text. Total and percent correct values were generated with this instrument. The total number of correct responses, out of eight, was denoted as TA.

Form F was used to gather data on the participant's overall ability to provide a complete solution to each randomly selected AWP. Table A1 (Appendix A) provides a master list of 15 AWP. Information gathered during the pilot study prompted the researcher to reduce the number of usable AWP to seven. The participant worked four randomly selected AWP from the seven selected AWP (Appendix B). Each participant solution was compared to the researcher-prepared solutions and scored using Form G, Student AWP Solution Scoring Rubric. A three- and four-point Likert-type scale was used for scoring. Ten values were generated for each of the student solutions. Qualitative and quantitative ordinal-level data were generated from the scoring of the student solutions. The mean of the four scores for *correct solution* was calculated and is denoted as CS.

Parents were given an Informed Consent Form for completion. The participants were given an Assent Form for completion, and a debriefing was conducted at the end of the assessment period.

Pilot Study Data and Discussion

A pilot study was conducted on the main campus of a 4-year college located in the area during three days in late April and early May 2013 with the approval of the Institutional Review Board of that institution. Eighteen college students participated in the study in three separate groups of five to nine students per session. The study group was comprised of eleven females and seven males, ranging in age from 18 to 30. There were 10 African-American, two Hispanic, and six Caucasian students. There were three

freshmen, four sophomores, nine juniors, and two seniors by college classification. There were no ESL identified students. The students were asked to identify their high school population size; using *small* as 0-500, *medium* as 501-1000, and *large* as 1001 or more students; two were from small schools, nine were from medium schools, and seven were from large schools. All 18 of the participants had completed Algebra I and II and Geometry in high school mathematics preparation, with six of them also having taken Calculus. All participants completed a modified version of Form A, *Pilot Study Student Information Form for Research Participation*, and Forms B, D, E, and F, to provide assessment data for evidence of Factor 2, 4, and 6 skill levels (Appendix C). All participants were given six randomly assigned AWP for completion from the 15 types in Table A1 (Appendix A).

Information gathered during the pilot study provided additional insight into the complexity of the AWP statements beyond reading difficulty level. With respect to identifying textual clues indicative of mathematical operations, participants correctly identified only 48.60% of all operations expressed or implied within the AWP statements. A review of the AWP statements revealed that explicit references to operations, such as *less than* or *more than*, occurred in only five of the 15 AWP. Similarly, explicit relational statements, such as *one of the integers is two less than the other*, occurred in only five of the 15 AWP statements. With respect to recognizing relational statements, participants correctly identified only 41.12% of all relations expressed or implied within the AWP statements. With respect to translating text into equations, participants wrote correct equations for only 30.53% of all AWP statements. Consequently, only seven of the original 15 AWP were used during the formal research study data collection. The seven problems were selected based on similarities of AWP statements and the

assessment statements in Forms C, D, and E.

The research design goal was to make possible the determination of the rank-order correlation between participant performance on the Form C, D, and E assessments and subsequent complete AWP solutions submitted on Form F. Spearman Rank-order Correlation Coefficients, r_s , based on comparisons between OA and CS, RA and CS, and TA and CS, as determined from the pilot study, are provided in Table 2.. The table also provides the *Percent Correct* for student performance on Forms C, D, E, and F. The Mathematical Operation Identification correlation of 0.329 was non-significant, the Recognizing Relational Statements correlation of 0.410 was non-significant, and the Translating Written Text into Mathematical Sentences correlation of 0.253 was non-significant, with $p > 0.10$ for all three comparisons.

Table 2

Spearman Rank-Order Correlation Coefficients for Comparisons of OA, RA, and TA with CS Values: Pilot Study Data

<i>Factor</i>	Correlations for <i>Factor</i> and CS comparisons	Percent Correct on Forms C, D, E: Factor Assessments	Percent Correct on Form F: AWP Solutions
2: Operation Identification, OA	0.329 (ns) $p > 0.10$	38.10%	48.60%
4: Recognizing Relations, RA	0.410 (ns) $p > 0.10$	58.73%	41.12%
6: Translating Equations, TA	0.253 (ns) $p > 0.10$	86.81%	30.53%

Research Design

Research participant data was collected using six instruments. Participant data was compiled on Forms A, B, C, D, E and F (Appendix C). The blend of qualitative and quantitative data identified the study as a mixed-methods model. The data collection measures were used to identify participant mathematical learning-style preferences; participant abilities related to Factors 2, 4, and 6; and AWP solution performance. The data gathered from Forms A and B was summarized into tabular form by response type. The participant responses for each AWP solved were scored from 0 to 2 (or 0 to 3). Higher scores were indicative of an above-average AWP solver, and lower scores were indicative of a below-average AWP solver.

The data gathered from Form C was summarized in tabular form, illustrated with simple graphs, and used to determine descriptive statistical measures such as the percent of questions correct. The data gathered from Forms D and E was treated in a similar fashion. Total and percentage correct scores on each of the seven AWP were calculated for each participant. The data from Forms C, D, E, and F were used to calculate Spearman's Rank-Order Correlations between total or percent correct for each participant, specific to each form. The variables were qualitative and quantitative, with nominal, ordinal, interval, and ratio-level data values, dependent upon the source of the data. Form F was used to record the participant's efforts to solve four randomly selected AWP from the modified group of seven AWP found in Appendix A (Table A4: *AWP Used in Data Collection, Modified from Table A1*). The seven AWP were subdivided into four groups based on common solution steps or problem types and the degree to which explicit statements were utilized in the AWP text. The participant was given one problem from each group, for which they were instructed to prepare a complete solution.

Further data were generated from the Form F solution efforts using the rubric found in Form G (Appendix D), as developed from the eight student factors mentioned within the research introduction. Each problem solution was assigned 10 scores based on how well the participant's prepared solution mirrored the researcher-prepared *ideal* solution.

Data Collection Procedures

The school district superintendent was contacted to secure permission to conduct the research within the district schools. Contact was subsequently made with high school principals to secure permission to conduct the research within the specific high schools. Contact was then made with individual classroom teachers who had agreed to assist with the research data collection. Appointments were made with each classroom teacher, suited to their instructional schedule, for a data-collection period. The data collection instruments had been revised after the pilot study to accommodate a 45 to 55 minute time period.

Upon identification and selection of an appropriate Algebra I, Algebra II or Pre-Calculus class, the classroom teacher completed Form A with the appropriate academic data for each participant to facilitate confidentiality of participant efforts through the use of a code number specific to each school and participant. Parental Consent Forms were sent home with the participant and collected by the classroom teacher. Participants who did not return the Consent Form were not included in the data-collection activities. Participants completed a Student Assent Form prior to participation in the study. The researcher collected the consent/assent forms from the classroom teacher along with Form A upon entering the classroom. The classroom teacher had previously discussed the data collection activity with the participants.

The classroom teacher introduced the researcher to the participants. The

researcher made a few prepared comments of greeting and appreciation for the opportunity to interact with the participants. Assessment packets prepared by the researcher were distributed to each participant. Each packet contained Forms B, C, D, E, and F in color-coded sheets for ease of identification during the procedures. Each participant was provided with several pencils, and calculator use was permitted only during completion of the Form F assessment. The researcher provided instruction on the objective and completion of Form B after having read the directions aloud. Participants completed the survey form, turned their packet to the next colored sheet, and waited until all other participants were finished. The researcher provided instruction on the objective and completion of Form C after having read the directions aloud. Participants completed the assessment form, turned their packet to the next colored sheet, and waited until all other participants were finished. The researcher provided instruction on the objective and completion of Form D after having read the directions aloud. Participants completed the assessment form, turned their packet to the next colored sheet, and waited until all other participants were finished. The researcher provided instruction on the objective and completion of Form E after having read the directions aloud. Participants completed the assessment form, turned their packet to the next colored sheet, and waited until all other participants were finished. The researcher provided instruction on the objective and completion of Form F after having read the directions aloud. Participants were allowed to use classroom calculators during this portion. Participants completed the assessment by providing a complete solution to each of four randomly selected AWP's, one solution per page. Participants raised a hand to indicate completion of the final assessment and their packet was collected.

The researcher provided comments of appreciation and read the Debriefing Form

aloud to the participants. Each participant was given a copy of the Debriefing Form. The researcher left the classroom.

Chapter 4: Results

Introduction

Descriptive statistics for participant group and subgroup demographics, rank-order correlation values for the three research questions, descriptive statistical values for participant subgroup assessment scores, one-way and two-way ANOVA analyses for subgroup mean comparisons, characteristics of proficient AWP solvers, and comparisons of participant subgroup Mathematical Learning-Style Survey responses were determined and are reported within this section.

Instrument Validation

Participant responses on Forms C, D, and E were used for determination of the variables OA, RA, and TA, which were in turn compared to the variable CS for the primary research question analyses. Two measures of internal consistency reliability were calculated. The Cronbach coefficient alpha was used in connection to Form C, which had 7 questions with 24 possible correct answers, not all dichotomous. The internal consistency reliability value for Form C was 0.817. The Kuder-Richardson 20 was used in connection with Forms D and E, which had 7 and 8 dichotomous questions, respectively. The KR-20 internal consistency reliability values were 0.675 and 0.500 for Forms D and E, respectively.

Descriptive Statistics for Participants

Descriptive statistics for participants were calculated and are shown in Table 3 indicating gender, ethnicity, grade-level, and course-enrollment subgroups. The Algebra I classes were populated by 9th graders only, the Algebra II classes were populated by 10th and 11th graders, and the Pre-Calculus classes were populated by 11th graders only. The gender percentages for females and males were 56.4% and 43.6%, respectively. The

ethnicity percentages for African American, Caucasian, and Hispanic were 28.2%, 63.8% and 8.0%, respectively. The grade-level percentages for 9th, 10th, and 11th were 62.5%, 11.7%, and 25.8%, respectively. The course enrollment percentages for Algebra I, Algebra II, and Pre-Calculus were 62.5%, 26.5%, and 11.0%, respectively. There were no 10th-grade Hispanic females and no 10th- or 11th-grade Hispanic males. The 10th-grade males were exclusively Caucasian.

The number and percentage composition of Algebra I participants (N=102) was determined from data provided in Table 3. The gender percentages for females and males were 51.0% and 49.0%, respectively. The ethnicity percentages for African American, Caucasian, and Hispanic were 35.3%, 52.9%, and 11.8%, respectively. As compared to the complete participant group, the percentages of African American and Hispanic participants were higher and the percentage of females was lower within the Algebra I group.

The number and percentage composition of Algebra II participants (N=43) was determined from data provided in Table 3. The gender percentages for females and males were 72.1% and 27.9%, respectively. The ethnicity percentages for African American, Caucasian, and Hispanic were 20.9%, 76.7%, and 2.3%, respectively. As compared to the complete participant group, the percentages of African American and Hispanic participants were lower and the percentage of females was higher within the Algebra I group.

The number and percentage composition of Pre-Calculus participants (N=18) was determined from data provided in Table 3. The gender percentages for females and males were 50.0% and 50.0%, respectively. The ethnicity percentages for African American, Caucasian, and Hispanic were 5.6%, 94.4%, and 0%, respectively. As

compared to the complete participant group, the percentages of African American and Hispanic participants were dramatically lower and the percentage of females was lower within the Pre-Calculus group.

Table 3

Participant Number and (Percentages) for Gender, Ethnicity, Course, and Grade Level

Course & Grade		Algebra I	Algebra II		Pre-Calculus	Sub-totals	Totals
Gender & Ethnicity		9 th	10 th	11 th	11 th		
Female	Afr Amer	20 (12.3%)	1 (0.6%)	5 (3.6%)	1 (0.6%)	27 (16.6%)	92 (56.4%)
	Caucasian	27 (16.6%)	9 (5.5%)	15 (9.2%)	8 (4.9%)	59 (36.2%)	
	Hispanic	5 (3.1%)	0	1 (0.6%)	0	6 (3.7%)	
Male	Afr Amer	16 (9.8%)	0	3 (1.8%)	0	19 (11.7%)	71 (43.6%)
	Caucasian	27 (16.6%)	9 (5.5%)	0	9 (5.5%)	45 (27.6%)	
	Hispanic	7 (4.3%)	0	0	0	7 (4.3%)	
Sub-totals	Afr Amer	36 (22.1%)	1 (0.6%)	8 (4.9%)	1 (0.6%)	46 (28.2%)	163 (100.0%)
	Caucasian	54 (33.1%)	18 (11.0%)	15 (9.2%)	17 (10.4%)	104 (63.8%)	
	Hispanic	12 (7.4%)	0	1 (0.6%)	0	13 (8.0%)	
Totals		102 (62.5%)	19 (11.6%)	24 (14.7%)	18 (11.0%)	163 (100.0%)	

Research Questions Data and Analyses

The data analyses procedures performed were divided into four sections: initially, to address the three research questions; secondly, to investigate potential effects of

gender, ethnicity, course, and grade level upon AWP solving performance; thirdly, to identify characteristics of a *proficient AWP solver* (PAWPS); and finally, to report on the strategies and practices reported by participants on Form B. The three research questions of this study were as follows:

1. To what degree does the student's ability to identify written clues indicative of mathematical operations impact AWP solution performance?
2. To what degree does the student's ability to recognize relational statements between component parts of the written text impact AWP solution performance?
3. To what degree does the student's ability to translate written text into mathematical equations impact AWP solution performance?

The participants' responses on Forms C, D, E and F provided the necessary data to investigate the three research questions. On Form C assessment, participants were required to identify the mathematical operations that would be used to write an equation to solve an AWP and provide the corresponding algebraic symbol, which would be used to solve seven AWP statements. The total number of correct responses out of 24 was recorded, denoted as variable OA. On Form D assessment, participants were required to identify sentences of the algebra word problem that indicated relationships between persons or objects mentioned in the problem, selecting the most appropriate mathematical expression or equation that represented a relational statement. The total number of correct responses out of seven was recorded, denoted as variable RA. On Form E, participants were required to read written statements to decide and select which of the equation choices responses was the most correct matching mathematical sentence. The total number of correct responses out of eight was recorded, denoted as variable TA. On Form F, participants were required to solve four AWP; three were randomly selected,

and one was assigned to all participants. Participants' individual AWP solutions were scored on 10 separate aspects of the work by using a researcher-prepared rubric, Form G. The rubric scoring was either a three-point or four-point Likert-type scale.

The mean of each participant's scores for the four AWP solutions was calculated and recorded, denoted as variable CS. Additionally, means were recorded for identifying operation clues, recognizing relational statements, and for the translation of text into equations. The mean of the four correct solution rubric scores the participant subgroups received for the AWP attempts on Form F was used to generate the statistics in Table 4. In addition, descriptive statistics on the number of correct responses on Forms D, E, and F were calculated and are provided in Appendix F (Tables F2, F3, and F4).

Table 4

Statistics for Complete Participant Group Performance on Initial Assessments and AWP Solution Efforts

Complete Participant Group, N=163	Mean	SD	Min	1st Qtr	Med	3 rd Qtr	Max	Poss. Range	Coef Var
OA: Number Correct on Form C	15.3	4.97	0	14	16	19	24	0-24	0.32
RA: Number Correct on Form D	2.7	1.30	0	2	3	4	6	0-7	0.48
TA: Number Correct on Form E	6.5	1.41	2	6	7	8	8	0-8	0.22
CS: Average Correct AWP Solution Scores, Form F	0.44	0.46	0	0	0.5	0.5	2	0-2	1.05

Rank-order Correlation Analyses

The research question analyses focused on determining measures of rank-order correlation between the participant's performance on Forms C, D, and E and the participant's performance on Form F, the complete AWP solution instrument. The

significance of each correlation was determined using standard statistical procedures. Spearman's Rank-Order Correlation Coefficient r_s was calculated rather than Person's r due to the ordinal nature of the Form F rubric scores. In order to address research question one, "To what degree does the student's ability to identify written clues indicative of mathematical operations impact AWP solution performance?" the means of the operations identification and correct AWP solution scores were analyzed (OA & CS), resulting in a significant rank-order correlation of $r_s=0.407$, $p<0.01$, $N=163$. The significance of the rank-order correlation coefficient indicates that low rankings associated with values of one variable tend to be paired with low rankings associated with values of the other variable, and pairings also occur in similar fashion for high rankings of each variable.

In order to address research question two, "To what degree does the student's ability to recognize relational statements between component parts of the written text impact AWP solution performance?" the means of the recognizing relations and correct AWP solution scores were analyzed (RA & CS), resulting in a significant rank-order correlation of $r_s=0.280$, $p<0.01$, $N=163$.

In order to address research question three, "To what degree does the student's ability to translate written text into mathematical equations impact AWP solution performance?" the means of the translating written text into equations and correct AWP solution scores were analyzed (TA & CS), resulting in a significant rank-order correlation of $r_s=0.169$, $p<0.05$, $N=163$.

The complete participant group was partitioned into eleven subgroups based on gender, ethnic, grade level, and course enrollment. There were four variable identifiers for each participant: gender, ethnicity, grade-level, and course enrollment. Only 9th

graders were enrolled in Algebra I, so those variables identified the same subgroup in all further analyses. The Pre-Calculus course contained only 11th graders. Algebra II was a mixture of 10th and 11th graders. The subgroups were categorized as 1-variable, 2-variable, 3-variable, and 4-variable and were analyzed separately.

The calculated r_s values for each 1-variable participant subgroup—classified by subgroup identifiers, subgroup sample size, and r_s values—are provided in Table 5. The final three rows of the table provide the number of cases and the percentage of separate analyses that were significant at either $\alpha=0.01$ or $\alpha=0.05$ levels, as well as the percentage of non-significant cases. The data within the table indicate significant rank correlations at either $\alpha=0.01$ or $\alpha=0.05$ levels between OA and CS for all subgroups except for Hispanic, 10th grade, and Pre-Calculus participants. Incidentally, these are the three smallest sample sizes within the 1-variable table. Significant rank-order correlations at either $\alpha=0.01$ or $\alpha=0.05$ levels between RA and CS exist for all subgroups, except for Male, African American, Hispanic, and Pre-Calculus. A significant rank-order correlation between TA and CS exists only for the Female subgroup.

Further analyses were conducted to calculate the rank-order correlations for other participant subgroups. Fifty-eight additional unique participant subgroups were identified and analyzed for significance of the rank-order correlations between OA and CS, RA and CS, and TA and CS. Results of the additional rank-order correlation analyses were provided in Appendix E (Tables E1, E2, and E3). Table E1 provided the statistical values obtained for 2-variable subgroups. The results were organized by the number of variables in the subgroupings, subgroup composition, and sample size in columns 1-3. Columns 4-6 contained the r_s values.

Table 5

Spearman's Rank-Order Correlations for ALL and 1-Variable Participant Subgroups

Subgroup	N	Participant Scores Analyzed, Rank-order Correlations		
		Operations Identification & Correct Solutions	Recognizing Relations & Correct Solutions	Translating Text & Correct Solutions
		(OA & CS)	(RA & CS)	(TA & CS)
ALL	163	0.407**	0.280**	0.169*
Female	92	0.460**	0.316**	0.212*
Male	71	0.337**	0.216	0.112
Afr Amer	46	0.504**	0.012	0.215
Caucasian	104	0.353**	0.377**	0.178
Hispanic	13	0.242	0.145	-0.262
9th/Alg I	102	0.374**	0.197*	0.110
10th grade	19	0.366	0.482*	-0.140
11th grade	42	0.422**	0.307*	0.296
Algebra II	43	0.384*	0.333*	0.085
Pre-Calculus	18	0.216	-0.294	0.307
Cases (%)	p<0.01	7 (63.6%)	3 (27.3%)	2 (18.2%)
	p<0.05	1 (9.1%)	4 (36.4%)	0
	NS	3 (27.3%)	4 (36.4%)	9 (81.8%)

Note. Correlations significant at $p < 0.01$ ** or $0.01 < p < 0.05$ *

Table E2 provided the statistical values obtained for 3-variable subgroups. The results were organized by the number of variables in the subgroupings, subgroup composition, and sample size in columns 1-4. Columns 5-7 contained the r_s values.

Table E3 provided the statistical values obtained for 4-variable subgroups. The results were organized by the number of variables in the subgroupings, subgroup composition, and sample size in columns 1-5. Columns 6-8 contained the r_s values. As can be seen in the last three rows of Table E3, the rank-order correlations between the participants' OA and CS values were significant at the 0.01 or 0.05 levels in 30.0% of all subgroups. The rank-order correlations between the participants' RA and CS values were significant at the 0.01 or 0.05 levels in 17.2% of all subgroups. The rank-order correlations between the participant's TA and CS values were significant at the 0.01 or 0.05 levels in 5.7% of all subgroups. The 30.0% OA and CS rank-order correlations are significant in nearly twice as many cases as the 17.1% RA and CS cases and more than five times as likely as in the TA and CS 5.7% cases. The 17.1% RA and CS rank-order correlations are significant in three times as many cases as the TA and CS 5.7% cases.

ANOVA Comparisons of Subgroup CS Means

A second aspect of the analyses focused on determining differences between mean values of CS for related subgroups using one-way ANOVA procedures. The participant group was divided using the four variables of gender, ethnicity, grade-level, and course enrollment, providing for four separate applications of the ANOVA procedure. Descriptive statistics for each of the eleven subgroups and specific to each of the four assessment scores--OA, RA, TA and CS--were calculated and are included in Appendix F (Tables F1, F2, F3, and F4). Data on the means is provided in Table F1. A one-way ANOVA comparing the CS scores of female and male participants was conducted with the results indicating no difference between gender with respect to ability to correctly solve AWP [$F(1,161) = 0.296, p > 0.10$]. A one-way ANOVA comparing the CS scores of African American, Caucasian, and Hispanic participants was conducted

with the results indicating no difference between the ethnic group participants with respect to ability to correctly solve AWP [$F(2,160) = 2.183, p > 0.10$].

A one-way ANOVA comparing the CS scores of 9th, 10th, and 11th grade participants was conducted with the results indicating a significant difference between the grade level participants with respect to ability to correctly solve AWP [$F(2,160) = 7.075, p < 0.001$]. Tukey's HSD was used to determine the nature of the differences between the grade levels. This analysis revealed that 9th graders scored lower ($m=0.336, sd=0.380$) than both 10th graders ($m=0.632, sd=0.503$) and 11th graders ($m=0.589, sd=0.538$). There was no significant difference between 10th and 11th grade participants.

A one-way ANOVA comparing the CS scores of Algebra I, Algebra II, and Pre-Calculus participants was conducted with the results indicating a significant difference between the different mathematics course participants with respect to ability to correctly solve AWP [$F(2,160) = 16.290, p < 0.001$]. Tukey's HSD was used to determine the nature of the differences between the courses. This analysis revealed that Pre-Calculus students scored higher ($m=0.944, sd=0.559$) than both Algebra I students ($m=0.336, sd=0.380$) and Algebra II students ($m=0.459, sd=0.440$). There was no significant difference between Algebra I and Algebra II student participants.

Two-way ANOVA procedures were performed to investigate potential interactions between participant characteristics as effecting overall AWP solution performance, for example, using gender and grade-level as the two factors and using CS as the dependent variable. All the two-way ANOVA calculations were determined to be non-significant.

Characteristics of a Proficient AWP Solver

The criteria for identification of a participant as a *proficient AWP solver (PAWPS)*

involved two characteristics. First, the participant's CS score must be in the top 20% of the complete participant group (n=163), and second, the participant must have used algebra processes in the solution of at least two of the four AWP's completed. Only 32 of the participants satisfied both criteria. Table 6 provides a statistical summary of the PAWPS subgroup, indicating gender, ethnicity, grade-level, and course enrollment. The PAWPS subgroup participants represented all genders, ethnicities, grade levels, and courses. The columns provide the variable identifier and CS scores from 2.00 to 1.00, along with the number of participants having that score. Variable totals and percentage of PAWPS group were calculated and shown in the last column. Examination of the percentages for the PAWPS subgroup as compared to the ALL participant group reveals no appreciable differences in gender percentage representation. Percentage representation is lower for African Americans, 9th graders, and Algebra I students. Percentage representation is higher for Caucasians, 10th graders, and Algebra II and Pre-Calculus students.

Table 6

CS Values for Proficient AWP Solver by Subgroups

N=32	Mean of AWP Correct Solution Scores, CS				PAWPS Total (Pct)	ALL Percent
	2.00	1.50	1.25	1.00		
Female		5	2	10	17 (53.1%)	56.44%
Male	2	2		11	15 (46.9%)	43.56%
Afr Amer		1	1	2	4 (12.5%)	28.22%
Cauc	2	6	1	17	26 (81.2%)	63.80%
Hispanic				2	2 (6.3%)	7.98%
9th		2	1	10	13 (40.6%)	62.58%
10th		3		3	6 (18.8%)	11.65%
11th	2	2	1	8	13 (40.6%)	25.77%
Algebra I		2	1	10	13 (40.6%)	62.58%
Algebra II		3		5	8 (25.0%)	26.38%
Pre-Calculus	2	2	1	6	11 (34.4%)	11.04%
Totals	2	7	2	21	32	

Statistics corresponding to the values of Table 4 were calculated for the PAWPS subgroup and are included in Table 7. The Table 4 values are repeated ALL data to facilitate comparisons between the PAWPS and ALL groups. The mean value for each of the five variables--OA, RA, TA and CS--were higher in the PAWPS subgroup than in the ALL group. The coefficient-of-variation values are lower for the PAWPS subgroup.

Table 7

Statistics for Proficient AWP Solver Performance on Initial Assessments and AWP Solution Efforts

		M	SD	Min	Q1	MD	Q3	Max	Poss. Range	Coef Var
OA: Number Correct on Form C	PAWPS	16.9	5.02	0	16	18	20	23	0-24	0.30
	ALL	15.3	4.97	0	14	16	19	24	0-24	0.32
RA: Number Correct on Form D	PAWPS	3.4	1.13	1	3	3.5	4	6	0-7	0.33
	ALL	2.7	1.30	0	2	3	4	6	0-7	0.48
TA: Number Correct on Form E	PAWPS	6.6	1.66	2	5.5	7	8	8	0-8	0.25
	ALL	6.5	1.41	2	6	7	8	8	0-8	0.22
CS: Average Correct AWP Solution Rubric Scores, Form F	PAWPS	1.2	0.30	1	1	1	1.5	2	0-2	0.25
	ALL	0.44	0.46	0	0	0.5	0.5	2	0-2	1.05

The five-number-summary values--minimum, 1st quartile (Q1), median, 3rd quartile (Q3), and maximum--were used to create box-and-whisker plots (or boxplots). The boxplots in Figure 2 indicate moderate differences between the two groups for each of the OA, RA, TA, and CS measures.

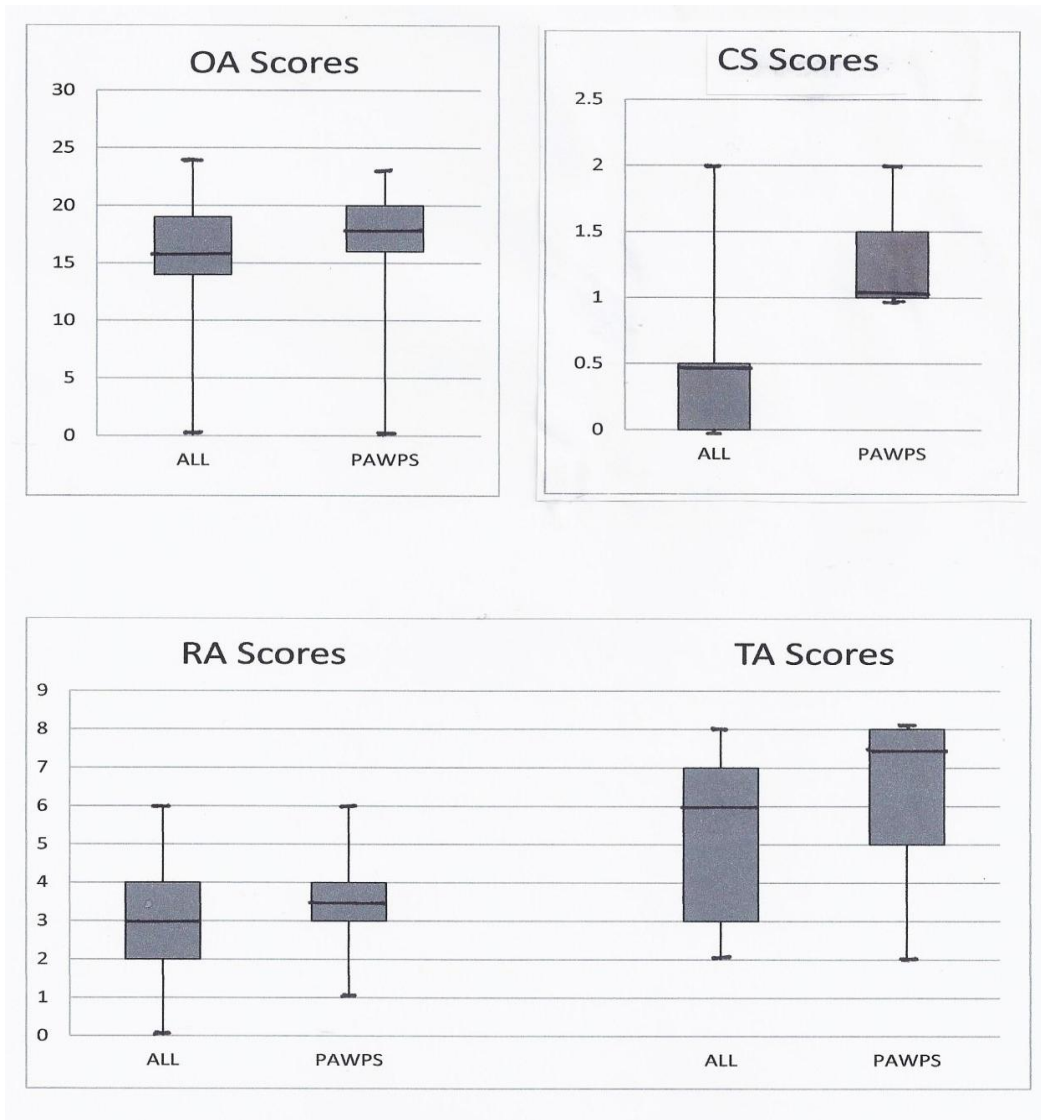


Figure 2. Boxplots of OA, RA, TA, and CS values for ALL and PAWPS subgroups

Table 8 provides information on the AWP solution strategies utilized by the PAWPS participants, the status of Yes or No for correct solution, and the CS values. A χ^2 hypothesis *Test for Independence of Variables* was conducted to determine the relationship between solution strategy employed and the CS score value. The test results were non-significant, with $\chi^2=28.885$, $df=18$, ($p=0.0812$), indicating that the CS value obtained was independent of the solution strategy employed. PAWPS participants used an *algebra-based solution* (Alg) in 23.4% of all AWPs attempted. A mixture of *algebra*

and *trial-and-error* (TeA) was used in 26.6% of attempts. *Trial-and-error* (TE) only was tried in 43.7% of all attempts. *No effort* occurred for 6.3% of AWP. Examination of the participants' AWP solution efforts during the scoring process revealed a tendency of the participant to begin the effort using algebra but to resort to trial-and-error to complete the work. Solution efforts for the more common sense questions, such as *Integer* and *Coin*, were predominantly trial-and-error.

Table 8

PAWPS Solution Strategy, Yes or No for Correct Solution, and CS Values

Solution Strategy	CS=2.00	CS=1.50	CS=1.25	CS=1.00	Totals (Pct)
Alg/Y	2	4	1	5	12 (9.3%)
Alg/N		4	2	12	18 (14.1%)
TeA/Y	6	8	1	18	33 (25.8%)
TeA/N				1	1 (0.8%)
TE/Y		9	3	20	32 (25.0%)
TE/N		1	1	22	24 (18.7%)
No Effort		2		6	8 (6.3%)
Totals (Pct)	8 (6.3%)	28 (21.9%)	8 (6.3%)	84 (65.5%)	128 (100%)

Table 9 provides information on the AWP solution strategies and the correct solution status for ALL and the PAWPS subgroup, based on the type of AWP being solved. Each participant was given three random AWP from three sets of two (*Integer* or *Sum*, *Coin* or *Cost*, and *Age* or *Rectangle*), and all participants were assigned the *Area* AWP. Totals and percentages were calculated. The data represents four AWP solutions for each of the N=163 participants for a total of 163 solutions. There were 4 times 32

AWP solutions for the PAWPS subgroup. The PAWPS subgroup had a higher number of correct solutions than the ALL group for all types of AWP, as seen in the *Correct* row of the summary section. As seen in the *Total* and *Pct* columns of the table, the PAWPS subgroup was able to obtain higher percentages of correct solutions and lower percentages of incorrect solutions for every type of strategy.

Table 9

AWP Solution Strategies and Correct Solution Status for ALL and PAWPS

AWP Soln Strategy/ CS Status	Group 652/128	Seven AWP Types Used in Data Collection								
		Intg	Sum	Coin	Cost	Age	Rect	Area	Total	Pct
Alg/Y	ALL	3	11	0	2	6	2	0	24	3.68
	PAWPS	1	4	0	2	3	2	0	12	9.45
Alg/N	ALL	23	10	3	11	34	19	20	120	18.40
	PAWPS	1	0	1	2	7	5	2	18	14.17
TA/Y	ALL	13	17	13	3	0	3	5	54	8.28
	PAWPS	8	7	8	3	0	3	4	33	25.98
TA/N	ALL	7	13	3	5	1	9	7	45	6.90
	PAWPS	0	0	0	0	0	0	1	1	0.79
TE/Y	ALL	24	9	19	2	0	1	12	67	10.28
	PAWPS	5	6	13	0	0	1	7	32	25.20
TE/N	ALL	10	14	31	29	24	27	48	183	28.07
	PAWPS	0	0	0	3	4	6	11	24	18.90
No Effort	ALL	3	6	14	28	20	17	71	159	24.39
	PAWPS	0	0	0	0	1	7	7	8	5.51
<u>Summary</u>										
Correct	ALL	40	37	32	7	6	6	17	145	22.24
	PAWPS	14	17	21	5	3	6	11	77	60.16
Incor/NE	ALL	43	43	51	73	79	72	146	507	77.76
	PAWPS	1	0	1	5	12	11	21	51	39.84

Participant Mathematical Learning Style Comparisons

Significant differences in CS values were determined for the comparisons

between PAWPS and ALL, between grade levels, and between course enrollments. Data was obtained from the participant responses on Form B and summarized in Appendix G (Tables G1, G2, and G3).

For the following discussion, *Non-PAWPS* refers to the 131 participants who were not classified as PAWPS. Table G1 contains data for comparison between the PAWPS and Non-PAWPS subgroups for each of the 17 questions asked on the form. The original survey contained 18 questions, but #17 was discarded after the pilot study, leaving only 17 questions. A chi-square test of independence was conducted for each of the 17 questions to determine the degree of independence between the PAWPS and Non-PAWPS subgroups regarding the responses provided. The hypothesis tests for independence of the variables produced significant results, $p < 0.05$, for questions #5 (checking answers), #6 (drawing a picture or sketch), #7 (personal rating of basic algebra equation solution ability), #8 (use of formula), #10 (use of correct geometric shape), #12 (experience with AWP), #13 (use of a chart or table), and #14 (identifying relations). The analyses were conducted to ascertain information on mathematical learning style characteristics of two groups, which might provide additional rationale for the differences in the two groups regarding AWP solution ability.

Similar analyses were conducted for grade-level comparisons, for the data in Table G2. The hypothesis tests for independence of the variables produced significant results, $p < 0.05$, for questions #5 (checking answers), #6 (drawing a picture or sketch), #7 (personal rating of basic algebra equation solution ability), #10 (use of correct geometric shape), #12 (experience with AWP), and #13 (use of a chart or table).

Similar analyses were conducted for course enrollment comparisons for the data in Table G3. The hypothesis tests for independence of the variables produced significant

results, $p < 0.05$, for questions #5 (checking answers), #6 (drawing a picture or sketch), #7 (personal rating of basic algebra equation solution ability), #10 (use of correct geometric shape), and #12 (experience with AWP).

Chapter 5: Discussion

Formal Research Questions Addressed

Mathematical literacy (ML) is a valued characteristic of individuals in a modern, global society. Commerce, industry, finance, banking, construction, and education are several key areas of human enterprise where the need for the consumer to be mathematically literate is paramount. ML is defined broadly in the International Life Skills Survey (ILSS) (2000) as “an aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work” (p. 12). As stated in *Principles and Standards for School Mathematics*, “Solving problems is not only a goal of learning mathematics but also a major means of doing so. . . . By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations” (NCTM, 2000, p. 52). The researcher has 45 years of experience in learning and teaching mathematics and has found in those years numerous examples of students struggling to solve problems of all types. AWP solutions were observed to be the most difficult for students, as borne out through the researcher’s extensive teaching experiences in secondary school and college algebra courses. The current research is the culmination of 45 years of wondering why students cannot solve AWP. AWP were defined within this study as *a problem statement consisting of one or more sentences having some known or unknown values, with explicitly or implicitly stated relationships between the values.*

The primary analyses focused upon determining the degree to which the participant’s performance in several preliminary tasks associated with AWP solution impacted the final result—the student obtaining the correct solution to the AWP. The

researcher suggested eight factors believed to affect student AWP solution performance, and three of these were the focus of the research. The three factors investigated within the research were: a) ability to identify written clues indicative of mathematical operations, b) ability to recognize relational statements between component parts of written text, and c) ability to translate written text into mathematical equations. Data on student performance in preliminary task assessments and complete AWP solution were collected through researcher-prepared instruments. Spearman's Rank-Order Correlation values were calculated for the pairwise comparisons of the following: a) OA and CS, b) RA and CS, and c) TA and CS, to address the primary research questions.

The first question related to identification of the operations either implied or explicitly stated in the AWP context, in order to compose one or more appropriate equations. The results were that 30.0% of all comparisons between the participant's performance in identifying mathematical operations and obtaining a correct AWP solution were significant at the $p < 0.05$ level. The interpretation is that, of the three current research questions, the ability of the participant to properly identify mathematical operations is the most important indicator of eventual success at correctly solving an AWP. Pape (2004) referred to student difficulties in interpreting operations based upon the use of *consistent* or *inconsistent language* in the AWP context, resulting in *reversal errors* (opposite operation used) and *mathematical errors* (misunderstanding of an operation). Although the current research did not specifically investigate the frequency of such errors, research participant performance on Form C for operations identification was 63.75% correct responses (mean=15.3 out of 24) and 38.10% correct responses for the pilot study data. The operations were limited to addition, subtraction, multiplication, and division. A typical algebra textbook contains an explanation of a word or phrase

indicative of mathematical operations, and Smith, Charles, Dossey, Keedy, and Bittenger (1990) concluded that students have a weakness in conceptual understanding of some operations; Alibadi et al. (2009) called for an increased focus upon operation sense, especially in the early grades. The students involved in the current research were in Grades 9 through 11. Sowder (1988) documented middle school student difficulties in identifying which operations should be used. The researcher concluded, based on evidence of previous studies and this research, that increased emphasis should be placed upon the teaching and practice of identifying operations as they are referenced in AWP statements at all grade levels where the student encounters such tasks.

The second question related to recognition of relational statements either implied or explicitly stated in the AWP context in order to compose one or more appropriate equations. The results were that 17.2% of all comparisons between the participant's performance in recognizing relations and obtaining a correct AWP solution were significant at the $p < 0.05$ level. Substantial research has concluded that students encounter difficulty in identifying and comprehending implied or explicitly stated relations and correctly representing relations in algebraic symbolism (Loftus, 1972; Polya 1957; Reed, 1987; Sternberg, 1985; Yerushalmy, 2006). Carraher (2006), in a study with fourth graders, determined that the students were able to understand relations between the processes expressed in story problems, a precursor to formal AWP. The researchers questioned how and why that early-onset skill at recognizing relations was lost and what instructional interventions might have taken place to prolong the knowledge and performance. Moseley and Brenner (2008) determined that 37% of university engineering students could not write a proper equation to represent and solve the statement "There are six times as many students as professors at this university" (p. 6).

This researcher did not explicitly investigate the occurrences of specific types of such relational recognition errors, such as *errors of commission* (Hall et al., 1989) and *problem representation errors* (Moseley & Brenner, 2008). Future research would supplement the existing knowledge along these lines of inquiry. Research participant performance on Form D for recognizing relations was 38.57% correct responses (mean=2.7 out of 7) and 57.83% correct responses for the pilot study data. The rank-order correlations for recognizing relations were significant in 17.2% of all comparisons and the rank-order correlations for identifying operations were significant in 30.0% of all comparisons. The smaller value of 17.2% suggested that, of the three research questions, student ability to recognize and represent relations is the second most important indicator of eventual success at correctly solving an AWP.

The third question related to translation of AWP text into equations, which would be solved using learned manipulative algebraic skills. The results were that 5.7% of all rank-order comparisons between the student's performance in translation of text into equations and obtaining a correct AWP solution were significant at the $p < 0.05$ level. The correlation value of 5.7%, when interpreted in reference to the earlier values of 30.0% for operations identification and 17.2% for recognizing relations, suggested that, of the three research questions, student ability to translate text into equations was the least important indicator of eventual success at correctly solving an AWP. No less than two-dozen references addressed the topic of *translation* of AWP text into symbolic algebraic form, specifically equations. Polya (1957), Kane (1970), and Mathews (1997) suggested that the student must first understand the verbal [written] version, a task that is complicated by the mixture of English language and mathematical language, in order to perform the very difficult task of translating from a written representation to a symbolic

representation. The studies referenced also suggested that the acquisition of the student's spoken or written language follows naturally with physical and intellectual growth; whereas, the mathematical language of symbolism and meanings does not develop without significant learning opportunities. Students typically have had years of natural language learning but experience an intense focus upon mathematical language (i.e., algebraic symbolism) for a few years during secondary school grades. Wollman (1983), Bernardo (1994), and Clement (1982) concluded that several types of errors commonly occur during the translation phase. A *reversal error* is the most common—the student tries to make the order of the algebraic symbolism match the word order in the text. A second common error is made when strictly following the syntactical structure of the text while ignoring the semantic meanings of the words relative to a mathematical context. Both errors were determined to be critically detrimental to the success of students in quantitative science courses and mathematics courses in secondary school and college. Clement (1982), Bernardo (1994), and Travis (1981) determined that students fail to recall the proper meanings of algebraic symbols, thus they use them incorrectly when translating text; this foundational misunderstanding provided for at least seven various types of errors associated with the translation phase. The seeming incongruity of the volume of research on the topic of translation and the research results from this study prompt the researcher to suggest that a more precise definition of *translation* be developed. The research efforts in this study did not attempt to identify the frequency of occurrence of the many types of translation-related errors. Research participant performance on Form E for translating text into equations was 81.25% correct responses (mean=6.5 out of 8) and 86.81% correct responses for the pilot study data. The higher percentage was likely due to the simple structure of the assessment questions. For Form

E the student read a single sentence and selected the corresponding equation from a list of choices. On Forms C and D, the student had to read multiple sentences and respond with correct choices, thereby making the task more difficult.

In summary of the three research questions, it is suggested that all three tasks—operations identification, recognizing relations, and actual translation of text into equations—are essential to the solution of an AWP. More research is warranted to better differentiate between the three tasks and to more fully understand the cognitive demands of each task in order to create instructional materials for student learning and practice in AWP solution procedures.

Discussion of ANOVA Results

The second part of the current research addressed the potential differences between genders, ethnicity groups, grade levels, and courses in the overall ability to correctly solve a selection of AWP, as measured by the variable CS, the mean score for four AWP solution efforts. A one-way ANOVA procedure was employed to analyze participant data. Four separate analyses were performed, one for each of the following comparisons: 1) female-male, 2) African American-Caucasian-Hispanic, 3) Algebra I-Algebra II-PreCalculus, and 4) 9th-10th-11th graders.

The results indicated no significant differences between gender in the ability to correctly solve AWP at $\alpha=0.05$ level of significance. Fennema (1981) concluded that females outperformed males in numeration skills at earlier ages but typically fall behind in performance in higher-level mathematics classes. The current research does not support that conclusion with respect to the solution of AWP. Data in Tables F1, F2, F3, and F4 (Appendix F) indicated that female and males differed by no more than 0.1 on the OA, RA, and TA means and by only 0.02 on the CS means. Females (N=10) and males

(N=11) were nearly equally represented in the PAWPS subgroup, with mean CS scores of 1.18 for females and 1.20 for males. Additionally, in the Spearman Rank-Order Correlation comparisons, females were included in 15 of the 66 (22.73%) significant cases; whereas, males were included in only 4 of the 66 (6.06%) significant cases, as determined from data in Tables 8, E1, E2, and E3 (see Appendix E). Halpern, as cited in Rathus (2010), indicated that females surpass males in verbal ability throughout their lives, possibly accounting for improved scores on AWP tasks. Current research data on reading comprehension scores for females and males extracted from Form F results indicated equal means of 1.32 on a scale of 0-3. The purported female advantage may be negated by the overall difficulty of the AWP solution task.

The results indicated no significant differences between ethnicity groups in the ability to correctly solve AWP at $\alpha=0.05$ level of significance. Despite the non-significant hypothesis results, African-American and Hispanic students have opportunities and challenges for growth in the task of learning to solve AWP, as their subgroup means were lower than the ALL means for each of the OA, RA, and CS measures as seen in Tables F1, F2, F3, and F4 (Appendix F). The TA mean for the African-American subgroup was also lower than the ALL mean. The means for the Caucasian subgroup were equal to or higher than the ALL means for each of the four measures. Bernardo (1999) found, “Students were better at comprehending the problem text when it was written in the student’s first and most proficient language” (p. 10). Abedi and Lord (2001) concluded, “English language learners [ELL] scored significantly lower than proficient speakers of English . . . and modifying the linguistic structure in math problems can affect student performance” (p. 231). Research data for the reading comprehension component of the AWP solutions indicated mean scores of 1.23 for

African-American, 1.34 for Caucasian, and 1.46 for Hispanic subgroups, out of a possible 0-3. Table A2 indicates the Flesch-Kincaid Grade Level Readability Scale (FKR) values for the seven AWP used in the data collection, and all of the AWP had an FKR value below 9.2. The potential issue of language-related difficulties was not addressed within the scope of the current research.

The results indicated a significant difference between grade-level groups in the ability to correctly solve AWP at $\alpha=0.05$ level of significance. The post-hoc Tukey HSD tests revealed significant differences for the 9th-10th grade comparison and the 9th-11th grade comparison, but no significant differences for the 10th-11th grade comparison. Given the significant results, 9th grade students have opportunities and challenges for growth in the task of learning to solve AWP, as their subgroup means were lower than the ALL means for each of the OA, RA, TA, and CS measures as seen in Tables F1, F2, F3, and F4 (Appendix F). The means for the 10th and 11th grade subgroups were higher than the ALL means for each of the four measures. Rasmussen and Marrongelle (2006), Yerushalmy (2006), and Weaver (1992) concluded that the ability of the student to successfully solve an AWP was directly impacted by the level of experience and degree of exposure that the student had in solving similar problems, and students will consequently develop knowledge appropriate to solving AWP over time. Data from Form B, question #12, was used to calculate the mean number of previous classes in which the participant had been exposed to AWP instruction. The means were 2.04 years, 2.89 years, and 3.21 years for the 9th, 10th, and 11th grade participant subgroups, respectively. Students having had more experiences with AWP are expected to be better at solving them.

Finally, the results indicated a significant difference between the course enrollees

in the ability to correctly solve AWP, $p < 0.05$. The post-hoc Tukey HSD tests revealed significant differences for the Algebra I and Pre-Calculus course comparison and the Algebra II and Pre-Calculus course comparison but no significant differences for the Algebra I and Algebra II course comparison. Given the significant hypothesis results, Algebra I course students have opportunities and challenges for growth in the task of learning to solve AWP, as their subgroup means were lower than the ALL means for each of the OA, RA, TA, and CS measures as seen in Tables F1, F2, F3, and F4 (Appendix F). The means for the Algebra II course subgroup were higher than the ALL means for each of the three preliminary assessment measures but lower than the CS value. Similar comments regarding experience are relevant to the rationale for course level differences but are not repeated here. The data collection was scheduled during the school calendar to follow the conclusion of the teaching of the chapter material for AWP solution procedures, and participants in the Algebra I classes had within the prior week completed study of AWP. The textbook material included discussion of similar AWP as were used in the research data collection, but participant AWP solution performance was poor to marginal, prompting a concern on the part of the researcher regarding the degree of emphasis being placed on the instruction of AWP. Vernooy (1997) commented,

Some students are going their entire grade school and high school careers without being required to learn to do word problems. Even their teachers tell them that ‘story problems are just too hard,’ or ‘that nobody can do word problems,’ and skip over the material. (p. 6)

In summary of the ANOVA procedures, non-significant results were found for both gender and ethnicity comparisons, and statistically significant results were determined for

both grade-level and course enrollment.

Discussion of the Characteristics of a Proficient AWP Solver

The selection of a participant to be designated as a *proficient AWP solver* (PAWPS) was based on two criteria: first, the participant's CS score must be in the top 20% of the complete participant group (N=163) and second, the participant must have used algebra processes in the solution of at least two of the four AWP's completed. Only 32 participants satisfied both criteria. Prior commentary has dealt with the advantage of experience as a factor determining potential AWP solving success. The data in Table 6 indicated that females and males were fairly equally represented among the PAWPS subgroup and in relative proportion to the ALL group for gender composition. African-American students were under-represented, Hispanic students were fairly equally-represented, and Caucasian students were over-represented, as compared to the ALL group composition. Ninth graders were under-represented, 10th graders were over-represented, and 11th graders were over-represented, in comparison to the ALL group. Algebra I students comprised the same subgroup as 9th graders, having a decrease. Algebra II students were slightly lower and Pre-calculus students were over-represented, as compared to the ALL group.

A second aspect of the PAWPS groups was determined from the performance values for the three assessments on Forms C, D, and E, as well as the AWP correct solution mean, CS. The PAWPS subgroup scored higher than the ALL group on all four measurements, OA, RA, TA, and CS, as indicated in Table 7. In particular, the coefficient of variation ($CV=SD/M$) for the PAWPS subgroup is lower than the comparable ALL measure. The boxplots in Figure 2 provided a visual of the differences between the ALL and PAWPS groups. For the OA and RA measures, the PAWPS upper

75% exceed the lower 50% of the ALL group. The difference was not as dramatic for the TA measure, with the PAWPS upper 75% exceeding approximately 40% of the ALL group. The most dramatic result is shown in the CS boxplots, as 100% of the PAWPS group scored higher than nearly 80% of the ALL group.

The third discussion topic concerned the strategies utilized by the PAWPS subgroup to solve the AWP. Specifically, the researcher considered whether the effort was exclusively algebra-based (ALG), a mixture of algebra and trial-and-error (TeA), entirely trial-and-error (TE), or no effort (NE) extended to solve the AWP. In addition, the researcher noted whether or not the participant obtained a correct solution to the AWP. Table 8 provided data on the CS score as related to the strategy employed and the correct/incorrect solution status indicated as a Y or N. As prescribed in the criteria for PAWPS selection, the student had to attempt an algebra-based solution for at least two of the four AWP attempts. The data indicated the use of algebra or TeA strategy in exactly 50.0% of the 128 attempts. A correct solution was obtained in 70.2% of those attempts. Although a reading of the seven selected AWP suggested that a TE solution strategy would probably be satisfactory, close examination of the solution efforts of 163 participants on four separate AWP ($4 \times 163 = 652$ AWP) indicated that the initial steps toward a TeA or TE solution quickly deteriorated into guessing at the solution. Examination of the data in Table 8 indicated that as reliance upon an algebra-based strategy lessened, then the likelihood of a higher CS value diminished. Calculations based on data for algebra or TeA solutions compared to TE or NE solutions provided that the algebra and TeA mean CS score was 1.27, compared to 1.11 for the TE or NE values. The values for CS were based on a 0-2 scale.

Table 9 demonstrated the differences between the ALL and PAWPS groups for

AWP solution strategy and Y or N correct solution as referenced by the actual problem being solved. The last column indicated that PAWPS students had a higher percentage of correct solutions and a lower percentage of incorrect solutions than the ALL group over all types of AWP. The summary values showed correct solutions by the PAWPS group in 60.16% of all AWP attempted and only 39.84% incorrect solutions. PAWPS were nearly three times as likely to obtain a correct solution ($60.16/22.24=2.71$) and obtained an incorrect solution in only one-half of their attempts ($39.84/77.76=1.95$).

In summary, the PAWPS subgroup outperformed the ALL group in nearly every comparison, yet the PAWPS subgroup was a suitable cross-section of the complete participant group.

Discussion of Participant Mathematical Learning Style Comparisons

The results of the ANOVA procedures indicated significant differences in CS values for the comparisons between PAWPS and non-PAWPS, between grade levels, and between course enrollments. The final discussion topic examined the participant responses on Form B, Mathematical Learning Style Survey (MLSS), in order to determine any experiences or strategies that positively impacted the AWP solution performance of the *significantly different groups*. Tables G1, G2, and G3 (Appendix G) provided data on the responses of the participants, separated by subgroup identifier, for the 17 questions on the MLSS. The results of the chi-square Test of Independence conducted for the three comparisons were previously mentioned. Responses deemed relevant to the differences between the non-PAWPS and PAWPS groups were from questions #5, #6, #7, #8, #10, #12, #13, and #14. For #5, 65.6% of PAWPS indicated they *always or frequently* checked the answer they obtained to determine the correctness of the proposed solution, but only 41.2% of non-PAWPS reported the same. For #6,

46.9% of PAWPS reported they *always or frequently* draw a sketch or picture when solving an AWP, but only 20.6% of non-PAWPS reported the same. For #7, 93.75% of PAWPS students rated their basic equation solving ability as a 4 or 5 (0-5 scale), but only 69.47% of non-PAWPS students provided similar ratings. For #8, 90.63% of PAWPS reported the use of a formula as *always or frequently*, but only 64.89% of non-PAWPS reported similar responses. The importance of the formula use is that the formula becomes the pattern for the creation for the equation used to solve the AWP. Without the formula as a pattern, the student is guessing at the appropriate relationship between the static or dynamic quantities in the problem statement. Nathan et al. (1992) and Travis (1981) concluded that the ability to access relevant long-term memory, such as previously-worked problems, action schemata, and problem representation was crucial to the overall success in solving an AWP. Students are required to recall formulas for distance, concentrations, perimeter, area, etc. as AWP's are attempted. For #10, 90.63% of the PAWPS students *always or frequently* selected the appropriate geometric sketch when necessary, but only 76.34% of the non-PAWPS students reported the same. Battista (1990) suggested that spatial visualization is an important factor in geometry achievement and geometric problem solving for both males and females, but the genders did not differ in their use of geometric problem-solving strategies. This is supported by the inclusion of nearly equal percentages of males and females in the PAWPS subgroup. For #12, 68.75% of PAWPS reported that their currently enrolled course was the *third or fourth* time where AWP's were taught or discussed, but only 41.22% of non-PAWPS reported similar responses. For #13, 84.38% of PAWPS reported that they *always or frequently* used a chart or table to help solve an AWP, but only 55.73% of the non-PAWPS reported the same responses. For #14, 68.75% of PAWPS reported that they

find it *very easy or easy* to identify the relationships between different parts of the AWP, but only 51.91% of the non-PAWPS reported the same. The responses were recorded in Table 10.

Responses which were deemed relevant to the differences between the grade-level groups were from questions #5, #6, #7, #10, #12, and #13. The ANOVA procedures conducted on the grade levels indicated equality of 10th and 11th, with 9th being less than both 10th and 11th. Table 10 provided supplemental evidence for the ANOVA results as it was noted that all percentages for 9th graders were less than the percentages for 10th and 11th graders.

Responses deemed relevant to the differences between students enrolled in different courses were from questions #5, #6, #7, #10, and #12. The ANOVA procedures conducted on the course enrollees indicated equality of Algebra I and Algebra II, with both Algebra I and Algebra II being less than Pre-Calculus. Table 10 provided supplemental evidence for the ANOVA results as it is seen that all percentages for Algebra I are less than the percentages for Pre-Calculus. For # 7, solving basic algebra equations and #10, correct geometry shape, Algebra II was higher than Pre-Calculus but lower for the remaining comparisons. The issue was possibly confounded by the existence of both 10th and 11th graders in the Algebra II course.

Table 10

Comparison of Subgroup MLSS Responses, Indicated as Percents

Question	PAWPS vs Non PAWPS		9 th vs	10 th vs	11th	Alg I vs Alg II vs PreCalc		
#5 Always or Frequently check answers	65.6	41.2	36.3	63.2	28.6	36.3	73.0	61.1
#6 Always or Frequently draw picture or sketch	46.9	20.6	11.8	57.9	45.2	11.8	54.1	55.6
#7 Rate ability to solve basis alg eqs (4 or 5)	93.8	69.5	70.6	94.7	73.8	70.6	91.9	83.3
#8 Always or Frequently use a formula with AWP	90.6	64.9	na	na	na	na	na	na
#10 Always or frequently select correct geom shape	90.6	76.3	68.6	94.7	97.6	68.6	97.6	94.4
#12 Years of experience with AWP (3 or 4)	68.8	41.2	23.5	84.2	85.7	23.5	81.0	100.0
#13 Always or Frequently use table or chart	84.4	55.7	55.9	73.7	69.1	na	na	na
#14 Very easy or easy to identify relationships in AWP	68.8	51.9	na	na	na	na	na	na

Contributions to the Literature

The primary contribution of this research to the existing literature is the determination of two preliminary tasks, operations identification and recognition of relationships, which must be proficiently performed by the AWP solver within the overall translation phase. The research results suggested that both operations identification and recognition of relations were more highly and significantly correlated to overall AWP solution success than the basic translation task.

Secondly, the lack of significant correlations based on gender or ethnicity

supports the notion that all students are equally capable of success at solving AWP, given appropriate instruction and practice. Thirdly, characteristics and strategies that separate proficient and non-proficient AWP solvers were identified. The researcher suggests identifying and implementing modifications to AWP solving instruction in Algebra I and Algebra II with the goal of developing similar problem-solving characteristics in mathematics students.

Limitations

All 9th-grade Algebra I classroom teachers had covered at least one chapter on AWP solution methods prior to the research data collection. A primary limitation of the current research relates to the variety of AWP studied during the actual classroom instruction. The classroom teachers completed an informal questionnaire to determine which of the seven AWP had actually been taught within the chapter, and several teachers indicated a likelihood that their students would not be familiar with each and every specific AWP used for data collection. This fact may account for some lower estimates of AWP solving ability within the Algebra I/9th grade group. As indicated previously, Algebra II and Pre-Calculus participants performed better than Algebra I participants, having had more experience with AWP. Secondly, participant subgroups from neighboring states were not utilized due to time constraints and school supervisory personnel refusal to allow participation. Thirdly, beyond the three represented ethnic groups, there were no participants of other major ethnic groups.

Delimitations

The design of this study did limit the participant base to South Carolina public high school students in one school district, for voluntary and involuntary reasons, as indicated above. Although Algebra I instruction does occur for some advanced students

as early as eighth grade, the research included only those participants within senior high school mathematics classrooms from 9th grade and beyond. Participants in Calculus classrooms were not included due to the lack of similar AWP content within that course, per researcher teaching experience. Also, there was no effort to include exceptionally low-level mathematics ability participants, due to perceived deficiencies in basic mathematics ability, such as reading comprehension and computational proficiency. The reduction of the number of AWP types used in the research study from 15 to seven was done to eliminate the AWP types that had *implied* references to operations and relations. These two items were an essential piece of the data collection instruments and analyses; hence AWP without *explicit* references to operations and/or relations were not used.

Considerations for Future Research

The researcher encourages future studies by interested parties. One area for further study would be to investigate AWP solving proficiency for the other eight AWP types listed in Table A1, which were not used in the current study. Data on student ability to effectively handle implicit references within AWP context may further emphasize the efforts of the Common Core State Standards for English Language Arts. The current research suggests that additional work needs to be done in order to better prepare students for the tasks of understanding and handling written narratives within mathematical contexts. A second area for research could extend the current research to other major ethnic groups; interesting comparisons might be made between student proficiency for Factors 2, 4, and 6 for AWP types expressed in their native languages.

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Appendix A

Table A1: Examples of Typical AWP Taught in Algebra I and II

Table A2: Essential Skills and Procedures Used to Solve AWP

Table A3: Flesch Reading Ease Score Mapping Table

Table A4: AWP Used in Data Collection, Modified from Table A1

Table A1

Examples of Typical AWP Taught in Algebra I and II

Problem Label	Problem Type and Statement	Source
RECTANGLE	<u>Geometry Problem:</u> The perimeter of a rectangle is 52 inches. If the length is 10 inches less than twice the width, find its dimensions.	Bluman (2005, p. 245)
SUM	<u>Number Problems:</u> The sum of four consecutive odd numbers is 64. What are the numbers?	Sterling (2008, p. 163)
INTEGER	<u>Number Problems:</u> The product of two integers is 48, and one of the integers is two less than the other. What are the two integers?	Sterling (2008, p. 156)
AGE	<u>Age Problems:</u> A woman is 6 years older than 5 times her house's age. The sum of the owner's age and the house's age is 48 years. How old is the house?	Vernooy (1997, p. 103)
COIN	<u>Coin Problem:</u> Jenny's coin purse contains 28 coins, all nickels and quarters. The value of the coins is \$2.40. How many of each kind does she have?	Vernooy (1997, p. 16)
WIND	<u>Rate, Time and Distance Problems:</u> A jet plane traveled with the wind for 240 miles, then turned around and flew against the wind for 192 miles. The two parts of the trip took an equal amount of time. The speed of the jet in still air is 360 mph. What is the speed of the wind?	Original problem
DISTANCE	<u>Rate, Time and Distance Problems:</u> Bob McGorkle left for a bicycle trip at 8:00AM, cycling at 12mph. Penny Jarkle followed Bob 30 minutes later, leaving from the same point, and caught up with him at 10:30AM. How fast was Penny going?	Vernooy (1997, p. 22)
MULTI	<u>Multi-digit Integer Problems:</u> The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number will be 45 less than the original. What is the number?	Vernooy (1997, p. 19)

COSTS	<u>Costs of Different Items:</u> Tickets for a flight from Dallas to San Francisco are \$363 for adults and \$242 for children. A plane took off with a full load of 168 passengers, and the total ticket sales were \$57,717. How many adults and how many children were aboard?	Vernooy (1997, p. 17)
MIXTURE	<u>Mixture Problem:</u> How many pounds of \$6.00-per-pound nuts and how many pounds of \$4.50-per-pound nuts need to be combined to make 18 pounds of a nut mixture which is worth \$5.75 per pound?	Vernooy (1997, p. 18)
ALLOY	<u>Solution and Alloy Problems:</u> A chemical manufacturer wants to mix some 10% pesticide solution with some 2% pesticide solution, so that the resulting mixture is 500 gallons of a 4.5% pesticide solution. How much of the 2% pesticide solution and how much of the 10% pesticide solution should be used?	Vernooy (1997, p. 18)
INVESTMENT	<u>Investment Problems:</u> Emma Nemm has \$18,000. She invests part of her money at 7.5% and the rest at 9%. If her income for one year from the two investments was \$1,560, then how much did she invest at each rate?	Vernooy (1997, p. 102)
WORK	<u>Work Problems:</u> Tom, Dick, and Harry arrive early one morning at the job site and get ready to paint a huge, old, Victorian mansion. Tom, working by himself, could paint the whole house in 14 days. It would take Dick 10 days to do the job by himself. And Harry could do the job in 8 days. How long does it take for the three men do the job working together?	Vernooy (1997, p. 223)
TRIANGLE	<u>Geometry Problem:</u> Jet A leaves the airport at 4 p.m. traveling due east at 550 mph. Jet B leaves the same airport at 4 p.m. traveling due south at 480 mph. How far apart are the jets at 7 p.m.?	Sterling (2008, p. 255)
AREA	<u>Geometry Problem:</u> The area of a rectangle is 108 square feet. If the length is increased by 1 and the width is decreased by 1, the area will be 104 square feet. What are the dimensions of the rectangle?	Vernooy (1997, p. 21)

Table A2

Essential Skills and Procedures Used to Solve AWP

Problem Label	Essential Skills and Procedures										
	Flesch- Kincaid Grade Level readability score	Flesch Reading Ease score (1)	Reading comprehension (1)	Recognize & draw sketch or figure	Formula selection (2)	Identify mathematical operation clues	Table for organizing information	Determine relationships within problem	Write equation for text (3)	Equation solving	Solution checking
Rectangle	6.4	68.7	Std	Y	Y	Y	N	Y-e	Y-1	Y	Y
Sum	5.0	70.0	FE	N	N	Y	N	Y-e	Y-1	Y	Y
Integer	5.3	77.4	FE	N	N	Y	N	Y-e	Y-2	Y	Y
Age	2.4	95.1	VE	N	N	Y	N	Y-e	Y-2	Y	Y
Coin	4.6	76.5	FE	N	N	Y	N	Y-i	Y-2	Y	Y
Wind	3.9	89.3	E	Y	Y-m	Y	Y	Y-i	Y-1	Y	Y
Distance	5.4	77.1	FE	Y	Y-m	Y	Y	Y-i	Y-1	Y	Y
Multi	5.6	72.6	FE	N	N	Y	N	Y-e	Y-2	Y	Y
Costs	9.1	55.4	FD	N	Y-cs	Y	N	Y-i	Y-2	Y	Y
Mixture	14.1	47.7	D	N	Y-cs	Y	Y	Y-i	Y-2	Y	Y
Alloy	14.4	32.6	D	Y	Y	Y	Y	Y-i	Y-2	Y	Y
Investment	7.0	67.0	Std	N	Y-m	Y	Y	Y-i	Y-2	Y	Y
Work	4.6	86.0	E	N	Y-cs	Y	N	Y-i	Y-1	Y	Y
Triangle	4.1	86.2	E	Y	Y-m	Y	N	Y-i	Y-1	Y	Y
Area	5.4	77.7	FE	Y	Y-m	Y	N	Y-e	Y-2	Y	Y

Notes. (1) See Reading comprehension labels in Table 5.

(2) *cs*: a formula selection based on "common-sense; *m*: a common mathematical formula

(3) *e*: *explicitly stated* relationship; *i*: *implied* relationship

(4) *Y-x* refers to the need for *x* equations; i.e., Y-2 requires two equations

Table A3

Flesch Reading Ease Score Mapping Table

Score	Readability Level	Reading Comprehension
0-29	Very Difficult	VD
30-49	Difficult	D
50-59	Fairly Difficult	FD
60-69	Standard	Std
70-79	Fairly Easy	FE
80-89	Easy	E
90-100	Very Easy	VE

Table A4

AWP Used in Data Collection, Modified from Table A1

Problem Label	Problem Type and Statement	Source
RECTANGLE	<u>Geometry Problem</u> : The perimeter of a rectangle is 52 inches. If the length is 10 inches less than twice the width, find its dimensions.	Bluman (2005, p. 245)
SUM	<u>Number Problems</u> : The sum of four consecutive odd numbers is 64. What are the numbers?	Sterling (2008, p. 163)
INTEGER	<u>Number Problems</u> : The product of two integers is 48, and one of the integers is two less than the other. What are the two integers?	Sterling (2008, p. 156)
AGE	<u>Age Problems</u> : A woman is 6 years older than 5 times her house's age. The sum of the owner's age and the house's age is 48 years. How old is the house?	Vernooy (1997, p. 103)
COIN	<u>Coin Problem</u> : Jenny's coin purse contains 28 coins, all nickels and quarters. The value of the coins is \$2.40. How many of each kind does she have?	Vernooy (1997, p. 16)
COSTS	<u>Costs of Different Items</u> : Tickets for a flight from Dallas to San Francisco are \$363 for adults and \$242 for children. A plane took off with a full load of 168 passengers, and the total ticket sales were \$57,717. How many adults and how many children were aboard?	Vernooy (1997, p. 17)
AREA	<u>Geometry Problem</u> : The area of a rectangle is 108 square feet. If the length is increased by 1 and the width is decreased by 1, the area will be 104 square feet. What are the dimensions of the rectangle?	Vernooy (1997, p. 21)

Appendix B

Solutions to Algebra Word Problems

Solution to RECTANGLE Problem (A)

Geometry Problem:

The perimeter of a rectangle is 52 inches. If the length is 10 inches less than twice the width, find its dimensions.

The problem is about the dimensions of a rectangle.

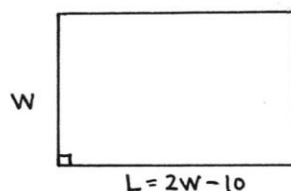
Let W = width
 L = length

Key Fact: perimeter of a rectangle

$$P = 2L + 2W$$

Relational equation:

$$L = 2W - 10$$



Solve:

$$52 = 2L + 2W$$

$$52 = 2(2W - 10) + 2W$$

$$52 = 4W - 20 + 2W$$

$$72 = 6W$$

$$12 = W$$

Now, $L = 2W - 10$

$$L = 24 - 10$$

$$L = 14$$

check: $52 = 2(14) + 2(12)$

$$52 = 28 + 24$$

$$52 = 52 \checkmark$$

Solution to SUM Problem

(B)

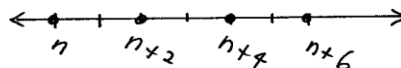
Number Problems:

The sum of four consecutive odd numbers is 64. What are the numbers?

The problem focuses on "consecutive odd" numbers.

(The student must recall that the sum of two odd numbers is even and that "consecutive odd" means the numbers "skip" some values.

Let n = smallest number
 $n+2$ = next odd number
 $n+4$ = third odd number
 $n+6$ = largest odd number



$$\begin{aligned} \text{Sum: } n + (n+2) + (n+4) + (n+6) &= 64 \\ 4n + 12 &= 64 \\ 4n &= 52 \\ n &= 13 \end{aligned}$$

$$\text{So, } n+2 = 15 ; n+4 = 17 ; n+6 = 19$$

$$\underline{\text{Check:}} \quad 13 + 15 + 17 + 19 = 64 \quad \checkmark$$

(Note: This is most likely an Alg I problem due to the single equation needed.)

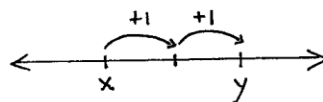
Solution to INTEGER Problem (C)

Number Problems:

The product of two integers is 48, and one of the integers is two less than the other. What are the two integers?

The problem is about operations on & relationships between two integers.

Let x = the larger integer
 y = the smaller integer } $x < y$



Product: $xy = 48$ Difference: $y - 2 = x$

Solve using substitution & factoring, if possible.

$$\begin{aligned} (y-2)y &= 48 \\ y^2 - 2y &= 48 \\ y^2 - 2y - 48 &= 0 \\ (y-8)(y+6) &= 0 \\ y &= 8, -6 \end{aligned}$$

Given a positive & negative value for y , two values have to be calculated for x .

$$\begin{aligned} y = 8 \text{ so } 8 - 2 &= 6 = x \\ \text{and} \\ y = -6 \text{ so } -6 - 2 &= -8 = x \end{aligned}$$

Denote (x_1, y_1) as $(-8, -6)$
 and (x_2, y_2) as $(6, 8)$

$$\begin{aligned} \text{Check: } (-8)(-6) &= 48 \checkmark \\ (-6) - (-8) &= 2 \checkmark \end{aligned}$$

$$\begin{aligned} (6)(8) &= 48 \checkmark \\ 8 - 6 &= 2 \checkmark \end{aligned}$$

Solution to AGE Problem

(D)

Age Problems:

A woman is 6 years older than 5 times her house's age. The sum of the owner's age and the house's age is 48 years. How old is the house?

This problem is about changing ages, especially "equal aging processes, where time moves the same for all."

$$\text{Woman} = 6 + (\text{House} + \text{House} + \text{House} + \text{House} + \text{House})$$

Let w = woman's age
 h = age of her house

Relational equations:

woman & house

$$w = 6 + 5h$$

total ages

$$w + h = 48$$

$$w = 48 - h$$

Solution: set w's equal to get

$$6 + 5h = 48 - h$$

$$6h = 42$$

$$h = 7 \text{ years}$$

$$\text{So, } w = 6 + 5h = 6 + 5(7) = 41 \text{ years}$$

$$\begin{aligned} \text{Check: } 41 &= 6 + 5(7) \\ 41 &= 41 \quad \checkmark \end{aligned}$$

Solution to COIN Problem

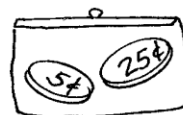
(E)

Coin Problem:

Jenny's coin purse contains 28 coins, all nickels and quarters. The value of the coins is \$2.40. How many of each kind does she have?

The problem is about coins: how many & how much money.

Let n = the number of nickels
 q = the number of quarters



The problem mentions total number of coins & total value of money, so two equations are needed:

$$\text{Coins: } n + q = 28 \qquad \text{Value: } 5n + 25q = 240 \text{¢}$$

Solve using one of three methods (graphing, substitution, or elimination).

Rearrange $n + q = 28$ to get $q = 28 - n$.

$$5(n) + 25(28 - n) = 240$$

$$5n + 700 - 25n = 240$$

$$-20n = -460$$

$$n = 23 \text{ (nickels)}$$

$$\text{So, } q = 28 - 23$$

$$q = 5 \text{ (quarters)}$$

Check: $n + q = 23 + 5 = 28 \quad \checkmark$

$$5(23) + 25(5) = 115 + 125 = 240 \text{¢} \quad \checkmark$$

Solution to WIND Problem

(F)

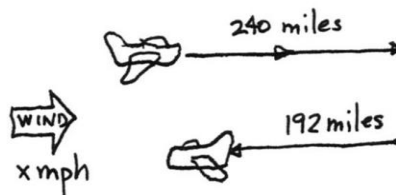
Rate, Time and Distance Problems:

A jet plane traveled with the wind for 240 miles, then turned around and flew against the wind for 192 miles. The two parts of the trip took an equal amount of time. The speed of the jet in still air is 360 mph. What is the speed of the wind?

This problem is about "motion aided & hindered" by natural forces such as the wind.

(Key Idea: rate of travel is increased 'with the wind' and decreased 'against the wind'.)

Key Fact: The plane flew for equal times, but unequal distances.



Formula: $D = RT$ or $T = \frac{D}{R}$

Let x = speed of the wind, in mph.

$360 + x$ = speed of plane 'with wind'

$360 - x$ = speed of plane 'against wind'

WIND	RATE	DISTANCE	TIME
with	$360 + x$	240 miles	$\frac{240}{(360+x)}$
against	$360 - x$	192 miles	$\frac{192}{(360-x)}$

Equal time equation:

$$\frac{240}{360+x} = \frac{192}{360-x}$$

$$240(360-x) = 192(360+x)$$

$$86,400 - 240x = 69,120 + 192x$$

$$17,280 = 432x$$

$$40 = x$$

mph

Check: $\frac{240}{400} = \frac{192}{320}$
 $0.6 = 0.6 \checkmark$

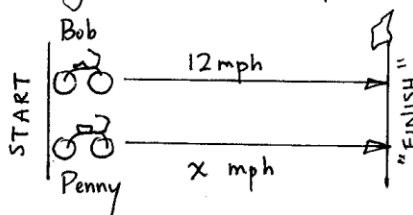
Solution to DISTANCE Problem (G)

Rate, Time and Distance Problems:

Bob McGorkle left for a bicycle trip at 8:00AM, cycling at 12mph. Penny Jarkle followed Bob 30 minutes later, leaving from the same point, and caught up with him at 10:30AM. How fast was Penny going?

The problem is about motion and "equal distance" travelled, for two persons moving at different speeds.

(The student must recall the formula for distance travelled $D = R \times T$.)



Key fact: Bob & Penny travel equal distances, but at different speeds & unequal times.

Fact Table: Let x = Penny's speed in mph.

	Start	End	Time	Rate	Distance
Bob	8 am	10:30	2.5 hr	12 mph	30 miles
Penny	8:30	10:30	2.0 hr	x	$2x$ miles

Since the distances travelled are equal
we have Bob's distance = Penny's distance

$$(12 \text{ mph})(2.5 \text{ hrs}) = (x)(2.0 \text{ hrs})$$

$$\frac{30 \text{ miles}}{2 \text{ hrs}} = x$$

$$15 \text{ mph} = x$$

Check: $(12)(2.5) = 30 = (15)(2) \checkmark$

Solution to MULTI Problem

(H)

Multi-digit Integer Problems:

The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number will be 45 less than the original. What is the number?

The problem is about reversal of digits of a number to get a new number. (The student must be able to express a multidigit number in terms of the separate digits.)

Let "tu" be the original number, so that the reversed digit number will be "ut".

In base 10, the value of a number = $10t + u$.

Equations:

Sum of digits:
in original $t + u = 11$

Difference between
original & reversed
digit number values:

$$\begin{aligned} (10t + u) - (10u + t) &= 45 \\ 9t - 9u &= 45 \\ t - u &= 5 \end{aligned}$$

Using elimination to solve

$$\begin{array}{r} t + u = 11 \\ + \quad t - u = 5 \\ \hline 2t = 16 \\ t = 8 \end{array}$$

$$\begin{aligned} \text{So, } 8 + u &= 11 \\ u &= 3 \end{aligned}$$

Original number "83"
Reversed number "38"

$$\begin{aligned} \text{Check: } 8 + 3 &= 11 \quad \checkmark \\ 83 - 38 &= 45 \quad \checkmark \end{aligned}$$

Solution to COSTS Problem

(I)

Costs of Different Items:

Tickets for a flight from Dallas to San Francisco are \$363 for adults and \$242 for children. A plane took off with a full load of 168 passengers, and the total ticket sales were \$57,717. How many adults and how many children were aboard?

The problem is about tickets & people on planes.

Let a = number of adults on board
 c = number of children on board



The problem mentions ticket prices, total number of tickets sold and the revenue generated. Two equations are needed:

$$\text{Tickets: } a + c = 168 \qquad \text{Revenue: } 363a + 242c = \$57,717$$

Solve using substitution, with $c = 168 - a$

$$363a + 242(168 - a) = 57,717$$

$$363a + 40,656 - 242a = 57,717$$

$$121a = 17,061$$

$$a = 141$$

$$\text{So, } c = 168 - 141 = 27$$

Check: $141 + 27 = 168 \checkmark$

$$363(141) + 242(27) = 51,183 + 6,534 = \$57,717 \checkmark$$

Solution to MIXTURE Problem

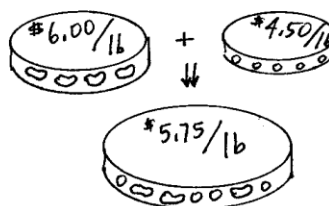
(J)

Mixture Problem:

How many pounds of \$6.00-per-pound nuts and how many pounds of \$4.50-per-pound nuts need to be combined to make 18 pounds of a nut mixture which is worth \$5.75 per pound?

The problem is about mixing different priced nuts to get a new blend.

Let s = number of pounds of \$6.00 nuts
 f = number of pounds of \$4.50 nuts



The problem mentions total number of pounds of \$5.75 nuts.
 No mention of total value of the new blend is given, but $(\text{pounds})(\text{unit price}) = (\text{total value})$

$$\text{Pounds: } s + f = 18 \quad \text{Value: } \$6.00s + \$4.50f = \$5.75(18) = \$103.50$$

Solve using elimination: $s + f = 18 \Rightarrow 6s + 6f = 108$

Eliminate $6s$ by subtraction:

$$\begin{array}{r} 6.00s + 6.00f = 108.00 \\ - 6.00s + 4.50f = 103.50 \\ \hline 0s + 1.50f = 4.50 \end{array}$$

$$f = 3$$

$$\text{So, } s = 18 - 3 = 15$$

Check: $s + f = 15 + 3 = 18 \checkmark$

$$6.00(15) + 4.50(3) = 90.00 + 13.50 = \$103.50$$

Solution to ALLOY Problem

(K)

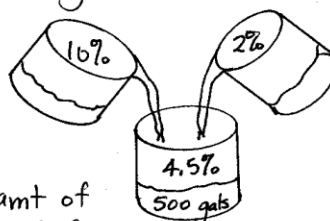
Solution and Alloy Problems:

A chemical manufacturer wants to mix some 10% pesticide solution with some 2% pesticide solution, so that the resulting mixture is 500 gallons of a 4.5% pesticide solution. How much of the 2% pesticide solution and how much of the 10% pesticide solution should be used?

The problem is about making a new solution which is more concentrated than one original part & less concentrated than the other original part.

(A key mixture principle is that the new concentration percent will be between the two original values.)

Let x = number of gallons of 10% solution
 y = number of gallons of 2% solution



Formula: $\frac{\text{solution volume}}{\text{concentration}} = \frac{\text{amt of pure substance}}{\text{percent}}$

The problem is solved by accounting for the total amount of pure substance, pesticide, in each solution.

Vol.	* Conc	= Pure
x	0.10	$0.10x$
y	0.02	$0.02y$
500 gal	0.045	22.50 gals

$$\begin{aligned} \text{Now, } x + y &= 500 \text{ so, } y = 500 - x \\ \text{and } 0.10x + 0.02y &= 22.50 \\ 0.10x + 0.02(500 - x) &= 22.50 \\ 0.10x + 10 - 0.02x &= 22.50 \\ 0.08x &= 12.50 \end{aligned}$$

$$x = 156.25$$

$$y = 500 - 156.25 = 343.75$$

Check: $156.25 + 343.75 = 500 \text{ gals} \checkmark$

$$0.10(156.25) + 0.02(343.75) =$$

$$15.625 + 6.875 = 22.50 \text{ gals} \checkmark$$

Solution to INVESTMENT Problem

(L)

Investment Problems:

Emma Nemm has \$18,000. She invests part of her money at 7.5% and the rest at 9%. If her income for one year from the two investments was \$1,560, then how much did she invest at each rate?

The problem is about earned interest from multiple investments, at different rates of interest.

Key formula: $I = P \times R$
 interest = principal \times rate
 or

interest = investment \times rate of interest

$$\boxed{\text{interest at 7.5\%}} + \boxed{\text{interest at 9\%}} = \boxed{\text{total interest}}$$

Rate	\times Investment	= Interest	
0.075	x	0.075x	
0.090	y	0.090y	
x	\$18,000	\$1,560	Total

Equations: Investment

$$\begin{aligned} x + y &= \$18,000 \\ y &= 18,000 - x \end{aligned}$$

Interest

$$0.075x + 0.09y = \$1,560$$

Substituting for y:

$$\begin{aligned} 0.075x + 0.09(18,000 - x) &= 1,560 \\ 0.075x + 1,620 - 0.09x &= 1,560 \\ -0.015x &= -80 \end{aligned}$$

Check: $4000 + 14000 = 18000 \checkmark$

$$\begin{aligned} 0.075(4000) + 0.09(14000) &= \\ 300 + 1260 &= 1560 \checkmark \end{aligned}$$

$$\begin{aligned} x &= \$4000 \\ y &= \$14,000 \end{aligned}$$

Solution to WORK Problem

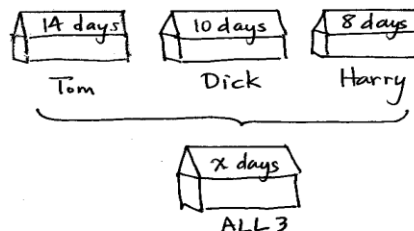
(M)

Work Problems:

Tom, Dick, and Harry arrive early one morning at the job site and get ready to paint a huge, old, Victorian mansion. Tom, working by himself, could paint the whole house in 14 days. It would take Dick 10 days to do the job by himself. And Harry could do the job in 8 days. How long does it take for the three men do the job working together?

This problem is about combined effort to complete "one job".

Key idea: all workers spend "equal" time at the task, even though they work at different rates, and the time spent will be less than the time of the "fastest painter."



Formula: work done =
rate \times time
 $W = RT$

All three work together, so the work done will be "one job."

$$\text{Tom's work} + \text{Dick's work} + \text{Harry's work} = 1$$

$$\left(\frac{1}{14}\right)x + \left(\frac{1}{10}\right)x + \left(\frac{1}{8}\right)x = 1$$

Each rate is the reciprocal of the time to do the job alone.

(Find a LCD (14, 10, 8) = 280)

$$\frac{20x}{280} + \frac{28x}{280} + \frac{35x}{280} = 1$$

$$73x = 280$$

$$x = 3.373 \text{ days}$$

$$\text{Check: } \left(\frac{1}{14} + \frac{1}{10} + \frac{1}{8}\right)(3.373) = \frac{73}{280}(3.373) = 1 \quad \checkmark$$

Solution to TRIANGLE Problem

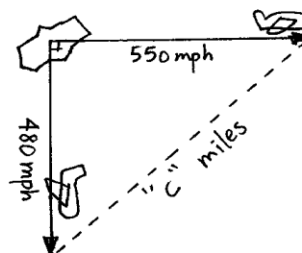
(N)

Geometry Problem:

Jet A leaves the airport at 4pm traveling due east at 550 mph. Jet B leaves the same airport at 4pm traveling due south at 480 mph. How far apart are the jets at 7pm?

The problem is a "motion problem" to determine total distance apart of moving objects.

(The student must recall compass directions, E&S as 90° apart, and the dynamic & changing distance between moving objects)



Recall two formulas:

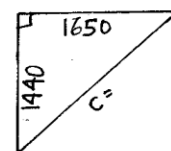
a) Distance = Rate \times Time
 $D = RT$

b) Pythagorean Theorem
 $a^2 + b^2 = c^2$

Each plane has flown for 3hrs (7pm - 4pm) at the time they are at the specific positions.

Jet A's distance = $(550 \frac{\text{miles}}{\text{hr}})(3 \text{ hr}) = 1650 \text{ miles}$

Jet B's distance = $(480 \frac{\text{miles}}{\text{hr}})(3 \text{ hr}) = 1440 \text{ miles}$



(The problem is more about motion dynamics & calculating an answer, than about "solving an equation".)

Solution: $a^2 + b^2 = c^2$

$$\sqrt{a^2 + b^2} = c$$

$$\sqrt{1650^2 + 1440^2} = c$$

$$2190 = c$$

miles

Solution to AREA Problem

(O)

Geometry Problem:

The area of a rectangle is 108 square feet. If the length is increased by 1 and the width is decreased by 1, the area will be 104 square feet. What are the dimensions of the rectangle?

The problem is about dimensions of a rectangle and how changing the length & width alter the area.

Let L = length of original rectangle
 W = width of original rectangle

$$\begin{array}{c} L \\ \boxed{w \quad A_{\text{orig}} = 108} \end{array}$$

so $L+1$ = new length
 $W-1$ = new width

$$\begin{array}{c} L+1 \\ \boxed{W-1 \quad A_{\text{new}} = 104} \end{array}$$

Recall area formula for rectangles:
 $A = LW$

Original area: $LW = 108$

so, $L = \frac{108}{W}$ use substitution & distributive property

New area: $(L+1)(W-1) = 104$

$$\left(\frac{108}{W} + 1\right)(W-1) = 104$$

$$\begin{aligned} 108 - \frac{108}{W} + W - 1 &= 104 \\ W - \frac{108}{W} &= -3 \end{aligned}$$

discard $W = -12$ as an impossible value.
 $L = \frac{108}{9} = 12$

(multiply by "W" and gather terms together as a quadratic)

$$W^2 + 3W - 108 = 0$$

$$(W+12)(W-9) = 0$$

$$W = -12, +9$$

Check: $(12)(9) = 108 \checkmark$
 $(13)(8) = 104 \checkmark$

Appendix C

Form A: School and Student Data Record Form for Research Participants

Form B: Student Mathematics Learning Style Questionnaire

Form C: Mathematical Operation Identification

Form D: Recognizing Relational Statements

Form E: Translating Written Text into Mathematical Sentences

Form F: Student AWP Solutions

Form B

Student Mathematics Learning Style Questionnaire

Directions: The following questions refer to your experiences in solving *algebra word problems* and *equation-only problems* in mathematics classes. Read each question statement carefully and select the response which most closely matches your mathematics learning style preference. Please circle the word or words shown in () which seem most like you or supply a number or letter when necessary.

1. I find *algebra word problems* to be (less, as, more) difficult to solve than a regular *equation problem*.
2. I read an *algebra word problem* (one, two, more than two) times before trying to solve the problem.
3. When I read an *algebra word problem*, I am (always, sometimes, never) able to identify the right math operation referred to in the problem.
4. Please rate your ability to correctly solve AWP, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good. _____
5. I (always, frequently, sometimes, never) check my answers to math problems to find out if they are correct.
6. I (always, frequently, seldom, never) draw a picture or sketch when trying to solve an *algebra word problem*.
7. Please rate your ability to correctly solve basic algebra equations, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good. _____
8. I (always, frequently, seldom, never) use a formula when solving an *algebra word problem*.
9. Learning mathematics is easier for me when I can _____ (select all that apply)
 - a. hear the words of the problem explained to me
 - b. see a picture, diagram, or sketch which illustrates the problem
 - c. use objects to count, build, or manipulate.

10. When an *algebra word problem* mentions a geometric shape- such as a circle, rectangle, or triangle, I (always, frequently, seldom, never) select the correct shape.
11. When reading and solving an *algebra word problem*, I find it (very easy, easy, difficult, very difficult) to write a correct equation to use to solve the problem.
12. My current math class is the (first, second, third, fourth) time that I have been enrolled in a class where *algebra word problems* are studied.
13. Using a table or chart to organize information (always, frequently, seldom, never) helps me solve an *algebra word problem*
14. When reading an *algebra word problem*, it is (very easy, easy, difficult, very difficult) for me to correctly identify the relationships between different parts of the problem.
15. When learning how to solve an *algebra word problem*, teachers recommended which of the following helpful strategies? (check all that apply)
- ___ a. draw a picture, sketch, or diagram ___ b. read the problem more than once
- ___ c. identify what values are known ___ d. choose a formula
- ___ e. identify what you want to know
- ___ f. select a variable to represent the unknown value(s)
16. When trying to solve an *algebra word problem*, I (never, seldom, often, always) use a *trial-and-error method* to calculate the number answer(s).
17. What step(s) in solving an *algebra word problem* do you find to be the most difficult?
18. When solving an *algebra word problem* I use (one, two) equations more often.

Form C

Mathematical Operation Identification

Directions: The following questions ask you to identify the mathematical operations which would be used to write an equation to solve an *algebra word problem*. Read each question statement carefully. Circle the word(s) or phrase(s) in the statement which help you decide and write the symbol for the operation below the words. DO NOT try to solve the problem.

1. One number is 5 times larger than another number. Their sum is 60.

What are the two numbers?

2. Josey worked a total of 39 hours in 1 week. She worked half as many overtime hours as she worked regular hours. How many overtime hours did she work?

3. After 7 weeks at the exercise club, Nancy could lift 70 pounds. This is 10 pounds more than twice what she could lift before joining the club. How much could Nancy lift before joining the club?

4. Alice lives 57 miles from Grandma's house, and Rex lives 83 miles from Grandma's house. How many fewer miles from Grandma does Alice live than Rex?

5. Two numbers are in a ratio of 4 to 7. The sum of the two numbers is 220. Find the numbers.

6. One side of a triangle is half the length of the longest side. The third side is 9 inches less than the longest side. The perimeter of the triangle is 196 inches. How long is each side?

7. A piece of pipe is 26 feet long. The pipe must be cut so that one piece is 6 feet shorter than the other. What are the lengths of the two pieces after it is cut?

Form D

Recognizing Relational Statements

Directions: The following questions require you to identify sentences of the *algebra word problem* which indicate relationships between persons or objects mentioned in the problem. You are to select the most appropriate mathematical expression or equation which represents a relational statement. Write the portion of the word problem which matches your choice, next to your response. Read each question statement carefully. DO NOT work the problem.

1. Michael is 3 years older than his brother Nick. In two years, he will be twice as old as Nick. How old is Nick? (pg 33, SWP)

- a) $M + 3 = N$
- b) $M + 2 = 2N$
- c) $M = N + 3$
- d) $M + 2 = 2N + 2$

2. Three consecutive odd integers add up to 759. What are the integers? (pg 50, SWP)

- a) $x + (x+1) + (x+2) = 759$
- b) $x + (x+2) + (x+4) = 759$
- c) $x(x+1)(x+2) = 759$
- d) $7 + 5 + 9 = x$

3. A dairy store sold a total of 80 ice cream sandwiches and ice cream bars. If the sandwiches cost \$0.69 each and the bars cost \$0.75 each and the store made \$58.08, find the number of each sold. (pg 112, MWPd)

- a) $S = 0.69$
- b) $0.69S + 0.75B = 80$
- c) $0.69S + 0.75B = 58.08$

4. The length of a rectangle is 5 feet more than the width. The perimeter of the rectangle is 58 feet. Find the width of the rectangle. (p538, ACS TB, #141)

- a) $W = L + 5$
- b) $L + W = 58$
- c) $LW = 58$
- d) $L = W + 5$

5. Peg walks 10 miles in the same time that Mae walks 6 miles. If Peg walks 1 mile per hour less than twice Mae's rate, what is the rate at which Peg walks? (p541, ASC TB, #177)

- a) $P + 10 = M - 6$
- b) $P = 2M - 1$
- c) $M = P - 10$
- d) $M = 2P + 1$

6. Judy has \$20,000 to invest. She plans to invest part at 5% in Bank A, with the remainder invested at 6% in Bank B. Find the amount invested at each rate if the total annual interest income is \$1060. (p539, ACS TB, #158)

- a) $A + B = 1060$.
- b) $20000 = 0.05A + 0.06B$
- c) $A + B = 20000$

7. The perimeter of a triangle is 70 centimeters. Two sides (A and B) of the triangle have the same length. The third side (C) is 7 centimeters longer than either of the equal sides. Find the length of the equal sides of the triangle. (p539, ACS TB, #151)

- a) $A + B - C = 70$
- b) $A + B + 7 = C$
- c) $C = A + 7$

Form E

Translating Written Text into Mathematical Sentences

Read each of the following written statements to decide and select which of the responses is the most correct matching mathematical sentence. DO NOT solve the problem.

1. The sum of a number and 7 is 76.

a) $n = 7 + 76$

b) $n + 7 = 76$

c) $n - 76 = 7$

2. If twice a number is decreased by 3 the result is 25.

a) $x - 3 = 25$

b) $x + 3 = 25$

c) $2x - 3 = 25$

d) $2x/3 = 25$

3. The quotient of a number and 7 is 6.

a) $n/7 = 6$

b) $7n = 6$

c) $n/6 = 7$

d) $7/n = 6$

4. Twice a number is 3 times the sum of the number and 7.

a) $2x + 3 = x + 7$

b) $2x = 3(x + 7)$

c) $3x + 7 = 2x$

5. 18 minus a number is equal to the number times 4.

a) $n - 18 = 4n$

b) $18 - n = 4n$

d) $4n = 18n$

6. The difference between 10 and a number is 7.

a) $x - 10 = 7$

b) $10 - x = 7$

c) $10x = 7$

d) $10/x = 7$

7. If twice a number is added to 40, the result is the number decreased by 7.

a) $2n + 40 - 7 = 0$

b) $40 + 2n = 2n - 7$

c) $40 + 2n = n - 7$

d) $n/2 + 40 = 7 - n$

8. 30% of the sum of a number and 8 is 5.

a) $0.30n + 8 = 5$

b) $0.30(n + 8) = 0.30(5)$

c) $0.30(n + 8 = 5)$

d) $0.30(n + 8) = 5$

Form F

Student AWP Solutions

Directions: You will be given four randomly chosen AWP to solve. You are to use a separate sheet of Form F for each problem. Prepare a complete solution for each problem. Show each solution on a separate answer sheet.

Appendix D

Form G: Student AWP Solution Scoring Rubric

Form G

Student AWP Solution Scoring Rubric

Directions: The rubric design is based on the eight student factors that are suggested in the current study as being essential to complete AWP solutions. Each participant solution effort will be scored using the rubric below. Note: Not every AWP has identical steps, but there are some commonalities. Scoring emphasis will be on completeness and correctness of the solution effort and on the existence of steps and procedures relevant to the three factors identified as research goals. The rubric score should reflect the extent to which the prepared solution evidences the student's ability, as measured by the eight factors. A four point Likert-type scale is used:

- 0 = no effort or no evidence of use of the factor in the solution process
- 1 = minimal effort or evidence of use, but with significant errors or misuse
- 2 = definite evidence of factor use, but with minor errors
- 3 = definite evidence of factor use, properly applied within correct solution

Student Ability Factor	Participant AWP Solution Scoring						
	Rectangle	Area	Integer	Age	Coin	Costs	Sum
1: Reading Comprehension							
2: Identifying Operation Clues							
3: Use of Sketch							
4: Recognizing Relational Statements							
5: Formula Selection							
6: Translation of Text to Equation							
7: Equation Solving							
8: Check Solution							
Trial and Error OR Algebra Solution							
Correct Solution							

Appendix E

Table E1: Rank-order Correlations for 2-Variable Participant Subgroups

Table E2: Rank-order Correlations for 3-Variable Participant Subgroups

Table E3: Rank-order Correlations for 4-Variable Participant Subgroups

Table E1

Rank-order Correlations for 2-Variable Participant Subgroups

			Participant Scores being Analyzed, Rank-order Correlation Coefs		
Subgroups		N	Operations Identification & Correct Solutions (OA & CS)	Recognizing Relations & Correct Solutions (RA & CS)	Translating Text & Correct Solutions (TA & CS)
F	A	27	0.651**	0.130	0.231
F	C	59	0.379**	0.397**	0.196
F	H	6	0.092	0.575	0.220
M	A	19	0.295	-0.144	0.189
M	C	45	0.344*	0.291	0.153
M	H	7	0.397	0.039	-0.546
F	N/O	52	0.400**	0.302*	0.108
F	T	10	0.470	0.303	0.264
F	E	30	0.489**	0.334	0.182
M	N/O	50	0.330*	0.068	0.106
M	T	9	0.254	0.645	-0.582
M	E	12	0.406	-0.083	0.611*
F	S	31	0.498**	0.234	0.333
F	P	9	-0.215	-0.155	0.064
M	S	12	0.219	0.523	-0.512
M	P	9	0.579	-0.570	0.585
A	N/O	36	0.540**	-0.078	0.108
A	E	9	0.279	0.093	0.452
C	N/O	54	0.270*	0.381**	0.154
C	T	18	0.294	0.409	-0.229
C	E	32	0.382*	0.277	0.262
H	N/O	12	0.279	0.075	-0.185
A	S	9	0.398	0.302	0.599
C	S	33	0.332	0.313	-0.021
C	P	17	0.195	-0.290	0.297
S	T	19	0.366	0.482*	-0.140
S	E	24	0.357	0.021	0.179
P	E	18	0.216	-0.294	0.307
Cases (%)	p<0.01	6 (21.4)		2 (7.1)	0
	p<0.05	4 (14.3)		2 (7.1)	1 (3.6)
	NS	18 (64.3)		24 (85.8)	27 (96.4)

Note: Correlations significant at $p < 0.01$ ** or $0.01 < p < 0.05$ *. The subgroup identifiers are F=Female, M=Male, A=African American, C=Caucasian, H=Hispanic, N/O=9th grade/Algebra I, T=10th grade, E=11th grade, S=Algebra II, and P=Pre-Calculus. A 2-variable example would be “Female and Hispanic”, denoted as *FH*.

Table E2

Rank-order Correlations for 3-Variable Participant Subgroups

				Participant Scores being Analyzed, Rank-order Correlation Coefs		
Subgroups			N	Operations Identification & Correct Solutions (OA & CS)	Recognizing Relations & Correct Solutions (RA & CS)	Translating Text & Correct Solutions (TA & CS)
F	A	N/O	20	0.618	-0.109	-0.002
F	A	E	6	0.726	0.302	0.726
F	C	N/O	27	0.257	0.539*	0.125
F	C	T	9	0.321	0.057	0.201
F	C	E	23	0.404	0.293	0.103
F	H	N/O	5	0.229	0.395	0.645
M	A	N/O	16	0.449	-0.012	0.260
M	C	N/O	27	0.292	0.134	0.172
M	C	T	9	0.254	0.645	-0.582
M	C	E	9	0.579	-0.570	0.585
M	H	N/O	7	0.397	0.039	-0.546
F	A	S	6	0.820*	0.470	0.826*
F	C	S	24	0.380	0.071	0.224
F	C	P	8	-0.346	-0.132	0.065
M	C	S	9	0.254	0.645	-0.582
M	C	P	9	0.579	-0.570	0.585
F	T	S	10	0.470	0.303	0.264
F	E	S	21	0.448*	0.090	0.220
F	E	P	9	-0.215	-0.155	0.064
M	T	S	9	0.254	0.645	-0.582
M	E	P	9	0.579	-0.570	0.585
A	E	S	8	0.173	-0.057	0.423
C	T	S	18	0.294	0.409	-0.229
C	E	S	15	0.343	-0.069	0.092
C	E	P	17	0.195	-0.290	0.297
F	A	N/O	20	0.618	-0.109	-0.002
F	A	E	6	0.726	0.302	0.726
F	C	N/O	27	0.257	0.539	0.125
Cases (%)			p<0.01	1 (4.0)	1 (4.0)	0
			p<0.05	2 (8.0)	0	1 (4.0)
			NS	22 (88.0)	24 (96.0)	24 (96.0)

Note: Correlations significant at $p < 0.01$ ** or $0.01 < p < 0.05$ *. The subgroup identifiers are F=Female, M=Male, A=African American, C=Caucasian, H=Hispanic, N/O=9th grade/Algebra I, T=10th grade, E=11th grade, S=Algebra II, and P=Pre-Calculus. A 3-variable example would be “Female, Caucasian and Eleventh,” denoted as *FCE*.

Table E3

Rank-order Correlations for 4-Variable Participant Subgroups

					Participant Scores being Analyzed, Rank-order Correlation Coefs		
Subgroups				N	Operations Identification & Correct Solutions (OA & CS)	Recognizing Relations & Correct Solutions (RA & CS)	Translating Text & Correct Solutions (TA & CS)
F	A	E	S	5	0.645	0.000	0.740
F	C	T	S	9	0.321	0.057	0.201
F	C	E	S	15	0.343	-0.069	0.092
F	C	E	P	8	-0.346	-0.132	0.065
M	C	T	S	9	0.254	0.645	-0.582
M	C	E	P	9	0.579	-0.570	0.585
Cases (%)				p<0.01	0	0	0
				p<0.05	0	0	0
				NS	6 (100.0)	6 (100.0)	6 (100.0)
Total Cases (Pct)				p<0.01	14 (20.0)	6 (8.6)	0
				p<0.05	7 (10.0)	6 (8.6)	4 (5.7)
				NS	49 (70.0)	58 (82.8)	66 (94.3)

Note: Correlations significant at $p < 0.01$ ** or $0.01 < p < 0.05$ *. The subgroup identifiers are F=Female, M=Male, A=African American, C=Caucasian, H=Hispanic, N/O=9th grade/Algebra I, T=10th grade, E=11th grade, S=Algebra II, and P=Pre-Calculus. A 4-variable example would be “Male, Caucasian, Tenth, and Algebra II”, denoted as *MCTS*.

Appendix F

Table F1: Statistics for Mean Correct Score *CS*, on Form F, by Participant Subgroups

Table F2: Statistics for Total Number Correct *OA*, on Form C, by Participant Subgroups

Table F3: Statistics for Total Number Correct *RA*, on Form D, by Participant Subgroups

Table F4: Statistics for Total Number Correct *TA*, on Form E, by Participant Subgroups

Table F1

Statistics for Mean Correct Score CS, on Form F, by Participant Subgroups

Subgroup	N	Pct	Mean	S.D.	Min	1st Quartile	Med	3 rd Quartile	Max
ALL	163	100.0	0.44	0.46	0		0.50		2.00
Female	92	56.4	0.42	0.44	0	0	0.50	0.50	1.50
Male	71	44.6	0.46	0.48	0	0	0.50	0.50	2.00
Afr Amer	46	28.2	0.32	0.35	0	0	0.25	0.50	1.50
Caucasian	104	63.8	0.49	0.50	0	0	0.50	0.94	2.00
Hispanic	13	8.0	0.42	0.33	0	0.13	0.50	0.50	1.00
Algebra I	102	62.6	0.34	0.38	0	0	0.25	0.50	1.50
Algebra II	43	26.4	0.46	0.44	0	0.25	0.50	1.00	1.50
Pre-Calculus	18	11.0	0.94	0.56	0	0	1.00	1.00	2.00
9th grade	102	62.6	0.34	0.38	0	0	0.25	0.50	1.50
10th grade	19	11.6	0.63	0.50	0	0	0.50	0.50	1.50
11th grade	42	25.8	0.59	0.54	0	0.50	0.50	1.31	2.00

Table F2

Statistics for Total Number Correct OA, on Form C, by Participant Subgroups

Subgroup	N	Pct	Mean	S.D.	Min	1st Quartile	Med	3rd Quartile	Max
ALL	163	100.0	15.3	4.97	0	14	16	19	24
Female	92	56.4	15.2	4.81	0	13.5	16	19	24
Male	71	44.6	15.3	5.20	0	13.5	16	19	22
Afr Amer	46	28.2	13.8	5.65	0	12	14	18	21
Caucasian	104	63.8	15.9	4.70	0	14	16	20	24
Hispanic	13	8.0	15.1	3.38	8	14	16	18	19
Algebra I	102	62.6	14.3	5.40	0	10	15.5	19	22
Algebra II	43	26.4	16.1	3.76	3	14	16	18	22
Pre-Calculus	18	11.0	18.4	2.94	13	16	18	20	24
9th grade	102	62.6	14.3	5.40	0	10	15.5	19	22
10th grade	19	11.6	17.7	2.77	11	16	18	20	22
11th grade	42	25.8	16.4	3.98	3	14	16	20	24

Table F3

Statistics for Total Number Correct RA, on Form D, by Participant Subgroups

Subgroup	N	Pct	Mean	S.D.	Min	1st Quartile	Med	3rd Quartile	Max
ALL	163	100.0	2.7	1.30	0	2	3	4	6
Female	92	56.4	2.7	1.32	0	2	3	4	6
Male	71	44.6	2.9	1.27	0	2	3	4	6
Afr Amer	46	28.2	2.5	1.39	0	2	2	4	6
Caucasian	104	63.8	2.9	1.26	0	2	3	4	6
Hispanic	13	8.0	2.4	1.12	0	2	2	3	4
Algebra I	102	62.6	2.6	1.21	0	2	2	3	6
Algebra II	43	26.4	2.7	1.46	0	2	3	3	6
Pre-Calculus	18	11.0	3.8	0.94	2	4	4	4	5
9th grade	102	62.6	2.6	1.21	0	2	2	3	6
10th grade	19	11.6	3.3	1.38	1	2.5	3	4	6
11th grade	42	25.8	2.8	1.43	0	2	3	4	6

Table F4

Statistics for Total Number Correct TA, on Form E, by Participant Subgroups

Subgroup	N	Pct	Mean	S.D.	Min	1st Quartile	Med	3rd Quartile	Max
ALL	163	100.0	6.5	1.41	2	6	7	8	8
Female	92	56.4	6.5	1.45	2	6	7	8	8
Male	71	44.6	6.5	1.36	2	6	7	8	8
Afr Amer	46	28.2	6.4	1.27	2	6	7	7	8
Caucasian	104	63.8	6.5	1.47	2	6	7	8	8
Hispanic	13	8.0	7.1	1.32	3	7	7	8	8
Algebra I	102	62.6	6.4	1.44	2	5	7	7	8
Algebra II	43	26.4	6.8	1.25	4	6	7	8	8
Pre-Calculus	18	11.0	7.0	1.46	2	7	7	8	8
9th grade	102	62.6	6.4	1.44	2	5	7	7	8
10th grade	19	11.6	7.1	1.22	4	6	8	8	8
11th grade	42	25.8	6.7	1.35	2	6	7	8	8

Appendix G

Table G1: Distribution of PAWPS and Non-PAWPS Participant Responses on Form B

Table G2: Distribution of Grade-level Participant Responses on Form B

Table G3: Distribution of Course Participant Responses on Form B

Table G1

Distribution of PAWPS and Non-PAWPS Participant Responses, Form B

Question Number and Statement	Subgrps	Response Level							
		0	1	2	3	4	5	6	7
1. I find <i>algebra word problems</i> to be (less, as, more) difficult to solve than a regular <i>equation problem</i>	PAWPS	2	10	20					
	Non-PAWPS	12	40	79					
2. I read an algebra word problem (one, two, more than two) times before trying to solve the problem.	PAWPS		1	13	18				
	Non-PAWPS		6	55	70				
3. When I read an <i>algebra word problem</i> , I am (always, sometimes, never) able to identify the right math operation referred to in the problem.	PAWPS	5	27						
	Non-PAWPS	9	117	4	1				
4. Please rate your ability to correctly solve AWP, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good.	PAWPS		1	5	15	10	1		
	Non-PAWPS	2	4	37	63	22	3		
5. I (always, frequently, sometimes, never) check my answers to math problems to find out if they are correct.	PAWPS	8	13	9	2				
	Non-PAWPS	16	38	67	10				
6. I (always, frequently, seldom, never) draw a picture or sketch when trying to solve an algebra word problem.	PAWPS	4	11	9	8				
	Non-PAWPS	6	21	63	41				
7. Please rate your ability to correctly solve basic algebra equations, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good.	PAWPS			1	1	20	10		
	Non-PAWPS		1	9	30	65	26		
8. I (always, frequently, seldom, never) use a formula when solving an algebra word problem.	PAWPS	8	21	13					
	Non-PAWPS	19	66	41	5				
9. Learning mathematics is easier for me when I can _ (select all that apply)		none	a	b	c	ab	ac	bc	abc
a. hear the words of the problem explained to me	PAWPS		4	8	1	12	1	4	2
b. see a picture, diagram, or sketch which illustrates the problem, c. use objects to count, build, or manipulate.	Non-PAWPS		26	27	8	31	8	13	18

Summary of #9 responses		a	b	c					
	PAWPS	19	26	18					
	Non-PAWPS	83	89	47					
10. When an algebra word problem mentions a geometric shape-such as a circle, rectangle, or triangle, I (always, frequently, seldom, never) select the correct shape.	PAWPS	15	14	3					
	Non-PAWPS	50	50	30	1				
11. When reading and solving an algebra word problem, I find it (very easy, easy, difficult, very difficult) to write a correct equation to use to solve the problem.	PAWPS	1	21	8	2				
	Non-PAWPS	4	58	62	7				
12. My current math class is the (first, second, third, fourth) time that I have been enrolled in a class where algebra word problems are studied	PAWPS		5	5	1 1	1 1			
	Non-PAWPS		27	50	3 5	1 9			
13. Using a table or chart to organize information (always, frequently, seldom, never) helps me solve an algebra word problem	PAWPS	4	23	5					
	Non-PAWPS	17	56	48	1 0				
14. When reading an algebra word problem, it is (very easy, easy, difficult, very difficult) for me to correctly identify the relationships between different parts of the problem	PAWPS	5	17	10					
	Non-PAWPS	1	67	59	3	1			
15. When learning how to solve an algebra word problem, teachers recommended which of the following helpful strategies? (check all that apply) ___ a. draw a picture, sketch, or diagram ___ b. read the problem more than once ___ c. identify what values are known ___ d. choose a formula ___ e. identify what you want to know ___ f. select a variable to represent the unknown value(s)		A	B	C	D	E	F		
	PAWPS	17	29	26	2 4	2 4	2 5		
	Non-PAWPS	63	12 1	10 1	6 6	8 7	9 2		
16. When trying to solve an algebra word problem, I (never, seldom, often, always) use a trial-and-error method to calculate the number answer(s).	PAWPS	7	13	12					
	Non-PAWPS	23	53	53	2				
18. When solving an algebra word problem I use (one, two) equations more often	PAWPS		21	11					
	Non-PAWPS		82	49					

		none	a	b	c	ab	ac	bc	abc
9. Learning mathematics is easier for me when I can _ (select all that apply) a. hear the words of the problem explained to me b. see a picture, diagram, or sketch which illustrates the problem, c. use objects to count, build, or manipulate.	9 th		25	20	7	20	7	10	13
	10 th			5	1	8		2	3
	11 th		5	10	1	15	2	5	4
Summary of #9 responses		a	b	c					
	9 th	65	63	57					
	10 th	11	18	6					
	11 th	29	34	12					
10. When an algebra word problem mentions a geometric shape- such as a circle, rectangle, or triangle, I (always, frequently, seldom, never) select the correct shape.	9 th	27	43	31	3				
	10 th	10	8	1					
	11 th	28	13	2					
11. When reading and solving an algebra word problem, I find it (very easy, easy, difficult, very difficult) to write a correct equation to use to solve the problem.	9 th	4	49	41	8				
	10 th		9	10					
	11 th	1	21	19	1				
12. My current math class is the (first, second, third, fourth) time that I have been enrolled in a class where algebra word problems are studied	9 th		30	48	12	12			
	10 th			3	15	1			
	11 th		2	4	19	17			
13. Using a table or chart to organize information (always, frequently, seldom, never) helps me solve an algebra word problem	9 th	9	48	39	6				
	10 th	1	13	2	3				
	11 th	11	18	12	1				
14. When reading an algebra word problem, it is (very easy, easy, difficult, very difficult) for me to correctly identify the relationships between different parts of the problem	9 th	2	52	45	2	1			
	10 th	2	9	8					
	11 th	2	24	15	1				
15. When learning how to solve an algebra word problem, teachers recommended which of the following helpful strategies? (check all that apply) ___ a. draw a picture, sketch, or diagram ___ b. read the problem more than once ___ c. identify what values are known ___ d. choose a formula ___ e. identify what you want to know ___ f. select a variable to represent the unknown value(s)		A	B	C	D	E	F		
	9 th	35	91	76	50	65	70		
	10 th	14	18	16	12	14	13		

	11 th	31	40	35	28	31	34		
16. When trying to solve an algebra word problem, I (never, seldom, often, always) use a trial-and-error method to calculate the number answer(s).	9 th	23	41	37	1				
	10 th	3	8	8					
	11 th	4	17	20	1				
18. When solving an algebra word problem I use (one, two) equations more often	9 th		60	12					
	10 th		14	5					

Table G3

Distribution of Participant Responses by Course, Form B

Question Number and Statement		Response Level							
		Alg I	Alg II	PreC					
1. I find <i>algebra word problems</i> to be (less, as, more) difficult to solve than a regular <i>equation problem</i>	Alg I	10	35	57					
	Alg II	3	10	24					
	PreC	1	5	12					
2. I read an algebra word problem (one, two, more than two) times before trying to solve the problem.	Alg I		5	41	56				
	Alg II			19	24				
	PreC		2	8	8				
3. When I read an <i>algebra word problem</i> , I am (always, sometimes, never) able to identify the right math operation referred to in the problem.	Alg I	9	89	3	1				
	Alg II	4	38	1					
	PreC	1	17						
4. Please rate your ability to correctly solve AWP, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good.	Alg I	1	4	29	45	20	3		
	Alg II	1	1	9	24	8			
	PreC			4	9	4	1		
5. I (always, frequently, sometimes, never) check my answers to math problems to find out if they are correct.	Alg I	8	29	60	5				
	Alg II	10	17	12	4				
	PreC	6	5	4	3				
6. I (always, frequently, seldom, never) draw a picture or sketch when trying to solve an algebra word problem.	Alg I	1	11	46	44				
	Alg II	6	14	19	4				
	PreC	3	7	7	1				
7. Please rate your ability to correctly solve basic algebra equations, using the numbers 0,1,2,3,4,5 with 0=very poor and 5=very good.	Alg I		1	8	21	57	15		
	Alg II			1	8	21	13		
	PreC			1	2	7	8		
8. I (always, frequently, seldom, never) use a formula when solving an algebra word problem.	Alg I	15	48	34	5				
	Alg II	10	25	8					
	PreC	2	7	7	2				
9. Learning mathematics is easier for me when I can		none	a	b	c	ab	ac	bc	abc

_ (select all that apply) a. hear the words of the problem explained to me b. see a picture, diagram, or sketch which illustrates the problem, c. use objects to count, build, or manipulate.	Alg I		25	20	7	20	7	10	13
	Alg II		5	9	1	17	2	3	6
	PreC			6	1	6		4	1
Summary of #9 responses		a	b	c					
	Alg I	65	63	57					
	Alg II	30	35	12					
	PreC	7	17	6					
10. When an algebra word problem mentions a geometric shape- such as a circle, rectangle, or triangle, I (always, frequently, seldom, never) select the correct shape.	Alg I	27	43	31	3				
	Alg II	26	16	1					
	PreC	12	5	1					
11. When reading and solving an algebra word problem, I find it (very easy, easy, difficult, very difficult) to write a correct equation to use to solve the problem.	Alg I	4	49	41	8				
	Alg II		24	18	1				
	PreC	1	6	11					
12. My current math class is the (first, second, third, fourth) time that I have been enrolled in a class where algebra word problems are studied	Alg I		30	48	12	12			
	Alg II		2	7	30	4			
	PreC				4	14			
13. Using a table or chart to organize information (always, frequently, seldom, never) helps me solve an algebra word problem	Alg I	9	48	39	6				
	Alg II	8	23	8	4				
	PreC	4	8	6					
14. When reading an algebra word problem, it is (very easy, easy, difficult, very difficult) for me to correctly identify the relationships between different parts of the problem	Alg I	2	52	45	2	1			
	Alg II	2	24	15	1				
	PreC	2	8	8					
15. When learning how to solve an algebra word problem, teachers recommended which of the following helpful strategies? (check all that apply) ___ a. draw a picture, sketch, or diagram ___ b. read the problem more than once ___ c. identify what values are known ___ d. choose a formula		A	B	C	D	E	F		
	Alg I	35	91	76	50	65	70		
	Alg II	32	42	35	27	30	31		

___ e. identify what you want to know ___ f. select a variable to represent the unknown value(s)	PreC	13	16	16	12	15	16		
16. When trying to solve an algebra word problem, I (never, seldom, often, always) use a trial-and-error method to calculate the number answer(s).	Alg I	23	41	37	1				
	Alg II	7	18	17	1				
	PreC		7	11					
18. When solving an algebra word problem I use (one, two) equations more often	Alg I		60	12					
	Alg II		31	12					